P.1: Algebraic Expressions and Real numbers

Algebraic Expression:

A combination of variables and numbers using the operation of addition $(+)$, subtraction$(-)$,multiplication$(\times)$ or division$(\div)$ as well as power or roots.

Example1:

$$x+6 \quad , \quad \frac{3x^2+2}{xy-3} \quad , \quad \sqrt{x-y}$$

Evaluating Algebraic Expression:

Order of Operations:

1- Perform operations within the **brackets** .
2- Evaluate all **Exponential expressions**.
3- Perform **Multiplication and division** (left to Right)
4- Perform **Addition and subtraction** (left to Right)

Example2: Evaluate Each Algebraic Expression for the given value of the variables:

$a)$ $x^2-3(x-y)$, for $x=8$ and $y=2$

$b)$ $\frac{2x+3y}{x+1}$ for $x=-2, y=4$

$c)\ 7 + 5x$ for $x = 10$

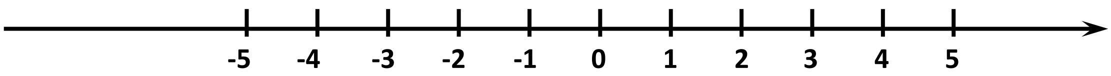
$d)\ 6x - y$ for $x = 3$ and $y = 8$

$e)\ x^2 - 6x + 3$ for $x = 7$

$f)\ 4 + 5(x - 7)^3$ for $x = 9$

$g)\ \frac{5(x+2)}{2x-14}$ for $x = 10$

The Real Number Line



Example3: Determine whether each statement is true or false:

$a)\ -13 < -2$

$b)\ \ 4 \geq -7$

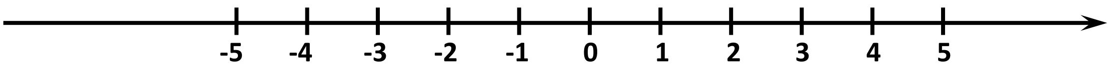
$c)\ -\pi \geq -\pi$

$d)\ 0 \geq -6$

Absolute Value:

The absolute value of a real number x, denoted by $|\boldsymbol{x}|$, is the distance from 0 to x on the number line. For example

$|-5| =$ $\quad$ $|-3| =$ $\quad$ $|0| =$ $\quad$ $|3| =$ $\quad$ $|5| =$



Definition of $|x|$ for any real number x

$$|x| = \begin{cases} x & if\ \ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

Note that: $|x| \geq 0$

Example4: Rewrite each expression without absolute value bars:

$\boldsymbol{a})\ \dfrac{\mathbf{-3}}{\mathbf{|-3|}}$

$\boldsymbol{b})\ \big||\mathbf{-3}| - |\mathbf{-7}|\big|$

Example5: Evaluate each algebraic expression for $x = 2$ and $y = -5$

$a)\, |x + y|$ $\qquad$ $b)\, |x| + |y|$ $\qquad$ $c)\, \dfrac{y}{|y|}$

Distance between points on real number line:

If $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two points on a real number line, then the Distance between two points $\boldsymbol{a}$ and $\boldsymbol{b}$ is given by

$$|\boldsymbol{a} - \boldsymbol{b}| = |\boldsymbol{b} - \boldsymbol{a}|$$

Example6: Express the distance between the given numbers **using absolute value**, then find the **distance** by evaluating the absolute value expression

$a)\ -2 \text{ and } 5$ $\qquad$ $b)\ -19 \text{ and } -4$

Simplifying Algebraic Expressions:

Order of Operations:

1- Perform operations within the **brackets**.
2- Evaluate all **Exponential expressions**.
3- Perform **Multiplication and division** (left to Right)
4- Perform **Addition and subtraction** (left to Right)

Example7: Simplify each Algebraic Expression:

a) $5(3y-2)-(7y+2)$

b) $5(3x-2)+12x$

c) $7(3y-5)+2(4y+3)$

c) $7-4[3-(4y-5)]$

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d) $8 - 2[3(2x - 5) - 9]$

e) $18x^2 + 4 - [6(x^2 - 2) + 5]$

f) $7x^2 - 3 - [2 - (x^2 - 1)]$

P.2: Exponents

Exponential Notation:

$$Base \leftarrow \boldsymbol{b}^{\boldsymbol{n} \rightarrow power\ or\ Exponent} = \boldsymbol{b.b.b}\ldots\ldots..\boldsymbol{b} \qquad \boldsymbol{n\ times}$$

$\boldsymbol{b^n}$ reads as " $\boldsymbol{b}$ to the power of $\boldsymbol{n}$" or " $\boldsymbol{b}$ to the $\boldsymbol{n}$th power".

Example 1:

A) $\mathbf{2^4} =$

B) $-\mathbf{2^4} =$

C) $(-\mathbf{2})^{\mathbf{4}} =$

D) $(-\mathbf{2})^{\mathbf{5}} =$

E) $-\mathbf{2^5} =$

Note: $\begin{cases} (-\boldsymbol{x})^{\boldsymbol{n}} = \boldsymbol{x}^{\boldsymbol{n}} & \textbf{if } \boldsymbol{n} \textbf{ is even number} \\ (-\boldsymbol{x})^{\boldsymbol{n}} = -\boldsymbol{x}^{\boldsymbol{n}} & \textbf{if } \boldsymbol{n} \textbf{ is odd number} \end{cases}$

Product Rule: $b^m b^n = b^{m+n}$

$a)$ $(-9x^3y)(-2x^6y^4) =$ $\qquad$ $\boldsymbol{b})$ $(11x^5)(9x^{12})$

Quotient Rule: $\dfrac{b^m}{b^n} = b^{m-n}$

$A)$ $\dfrac{x^{14}}{x^{-7}} =$ $\qquad$ $\boldsymbol{B})$ $\dfrac{20x^{24}}{10x^6} =$ $\qquad$ $B)$ $\dfrac{25a^{13}b^4}{-5a^2b^3} =$

P.2: Exponents

Zero –Exponent Rule: $b^0 = 1,\ b \neq 0$

$A)\ (-3)^0 =$ $\qquad$ $-9^0 =$

$B)\ -3^0 =$ $\qquad$ $(-9)^0 =$

Note: **<u>0^0 is undefined</u>**

Negative –Exponent Rule:

$$\boldsymbol{b^{-n} = \frac{1}{b^n}} \qquad \boldsymbol{\frac{1}{b^{-n}} = b^n}$$

$x^2y^{-3} =$ $\qquad$ $\dfrac{x^2}{y^{-3}} =$

Power Rule (Powers to Power): $\boldsymbol{(b^m)^n = b^{mn}}$

$\left(x^{-5}\right)^3 =$ $\qquad$ $(x^{11})^5 =$

Products to Powers: $\boldsymbol{(ab)^n = a^n b^n}$

$\left(-3x^2y^5\right)^2 =$ $\qquad$ $\left(4x^{-3}y^2\right)^{-2} =$

Quotients to Powers: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\left(-\frac{4}{x}\right)^3 =$ $\qquad$ $\left(\dfrac{3y^{-3}}{x^{-2}}\right)^2 =$

Simplifying Exponential Expressions:

Properties of exponents are used to simplify the exponential expressions. An exponential expression is simplified when

-No Parentheses appear.
-No powers are raised to powers.
-Each base occurs only ones.
-No negative or zero exponents appear.

Example2:

Simplify Each Exponential Expressions:

$A)\ x^{-5}.x^{10} =$

$B)\ \left(\frac{5x^3}{y}\right)^{-2} =$

$C)\ \left(-3x^4y^5\right)^3 =$

$D)\left(-7xy^4\right)\left(-2x^5y^6\right) =$

$E)\ -\frac{35x^2y^4}{5x^6y^{-8}} =$

$F)\ \left(\frac{4x^2}{y}\right)^{-3} =$

P.2: Exponents

$H)\ \left(\dfrac{4a^{-5}b^{3}}{12a^{3}b^{-5}}\right)^{0} =$

$I)\ \left(\dfrac{-20a^{3}b^{4}}{4a^{6}b^{-3}}\right)^{-2}$

$J)\ \left(\dfrac{-15a^{4}b^{2}}{5a^{10}b^{-3}}\right)^{3} =$

$K)\ \dfrac{24x^{3}y^{5}}{32x^{7}y^{-9}} =$

$L)\ \left(\dfrac{-30a^{14}b^{8}}{10a^{17}b^{-2}}\right)^{3} =$

$M)\ \left(\dfrac{x^{3}y^{4}z^{5}}{x^{-3}y^{-4}z^{-5}}\right)^{-2} =$

P.3: Radicals and Rational Exponents

Definition: *If* $\boldsymbol{a}$ *is nonnegative real number, the nonnegative number* $\boldsymbol{b}$ *such that* $\boldsymbol{b^2 = a}$, *denoted by* $\boldsymbol{b = \sqrt{a}}$, *is* ***principal square root*** *of* $\boldsymbol{a}$.

The sign $\sqrt{\ }$ is called Radical Sign, the expression $\sqrt{x}$ is called Radical Expression, the number inside the radical sign is called Radicand.

Example 1: Evaluate

$\sqrt{25} =$ $\qquad$ $\sqrt{64} =$ $\qquad$ $\sqrt{81} =$ $\qquad$ $\sqrt{16} =$

Undefined expressions: Expressions with negative radicands are undefined.

$\sqrt{\mathbf{-25}}, \sqrt{-4}, \sqrt{-16}$, and so on, are undefined.

Simplifying $\sqrt{a^2} = |x|$

For any real number a,

$$\sqrt{a^2} = |a|$$

Example 2: Evaluate

$\sqrt{(-13)^2} =$ $\qquad$ $\sqrt{(-3)^2} =$ $\qquad$ $\sqrt{(-5)^2} =$

The Product and quotient Rules for square Roots

If a and b are nonnegative real numbers and $b \neq 0$,then

1) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{a}\sqrt{b} = \sqrt{ab}$

2) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Example 3: Simplify the expressions; assume that the variables represent nonnegative number.

$A)\ \sqrt{x^3} = \sqrt{x^2 \cdot x} = \sqrt{x^2}\sqrt{x} = x\sqrt{x}$

$B)\ \sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

$C)\ \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

P.3: Radicals and Rational Exponents

$D)\ \sqrt{45x^2}$

$E)\ \sqrt{125x^2}$

$F)\ \sqrt{2x}\cdot\sqrt{6x}$

$G)\ \sqrt{10x}\cdot\sqrt{8x}$

$H)\ \sqrt{3x^2}\cdot\sqrt{6x} =$

$I)\ \dfrac{\sqrt{200x^3}}{\sqrt{10x^{-1}}} =$

$J)\ \dfrac{\sqrt{150x^4}}{\sqrt{3x}} =$

$K)\ \dfrac{\sqrt{48x^3}}{\sqrt{3x}} =$

P.3: Radicals and Rational Exponents

Adding and Subtracting Square Roots:

Two or more square roots can be added if they have the same radicand.

$$a\sqrt{x}+b\sqrt{x}=(a+b)\sqrt{x} \qquad a\sqrt{x}-b\sqrt{x}=(a-b)\sqrt{x}$$

Example 4: Add or subtract terms whenever possible.

1) $6\sqrt{17x}-8\sqrt{17x}=$

2) $3\sqrt{18}+5\sqrt{50}=$

3) $4\sqrt{12x}-2\sqrt{75x}=$

4) $\sqrt{50x}-\sqrt{8x}=$

5) $4\sqrt{12x}-2\sqrt{75x}=$

6) $3\sqrt{8}-\sqrt{32}+3\sqrt{72}-\sqrt{75}=$

Rationalizing denominators:

Rationalizing denominators means rewrite a radical expression as an equivalent expression in which the denominator no longer contains any radicals.

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \left(\frac{\sqrt{b}}{\sqrt{b}}\right) = \frac{a\sqrt{b}}{\sqrt{b}^2} = \frac{a\sqrt{b}}{b}$$

Example 5:

$A)\quad \frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{5}^2} = \frac{3\sqrt{5}}{5}$

$B)\quad \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

$C)\quad \frac{2}{\sqrt{6}} = \frac{2}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

Conjugate Expressions

Radical expressions that involve the sum and difference of the same two terms are called conjugates. Thus,

$\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are conjugates.

Conjugates are used to rationalize denominators because the product of such a pair contains no radicals:

$$\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = \sqrt{a}^2 - \sqrt{b}^2 = a - b$$

Example 6:

$\left(\sqrt{5} + \sqrt{2}\right)\left(\sqrt{5} - \sqrt{2}\right) = \sqrt{5}^2 - \sqrt{2}^2 = 5 - 2 = 3$

$\left(4 + \sqrt{7}\right)\left(4 - \sqrt{7}\right) = 4^2 - \sqrt{7}^2 = 16 - 7 = 9$

We can rationalize a denominator if the denominator contains two terms with one or more square roots by multiplying the numerator and the denominator by the conjugate of the denominator.

$$\frac{1}{\sqrt{a} + \sqrt{b}} = \frac{1}{\sqrt{a} + \sqrt{b}} \cdot \left(\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} - \sqrt{b}}\right) = \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a}^2 - \sqrt{b}^2} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$$

Example 7:

$$\frac{4}{\sqrt{6} + \sqrt{2}} = \frac{4}{\sqrt{6} + \sqrt{2}}\frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} = \frac{4\left(\sqrt{6} - \sqrt{2}\right)}{6 - 2} = \frac{4\left(\sqrt{6} - \sqrt{2}\right)}{4} = \sqrt{6} - \sqrt{2}$$

Example 8: Rationalize the denominator:

$A)\ \frac{\sqrt{2}}{\sqrt{5}} =$

$B)\ \frac{6}{\sqrt{5}+\sqrt{3}} =$

$C)\ \frac{13}{3+\sqrt{11}} =$

$D)\ \frac{3}{3+\sqrt{7}} =$

$E)\ \frac{\sqrt{2}}{\sqrt{12}-3} =$

$F)\ \frac{7}{\sqrt{5}-2} =$

Other Kinds of Roots: $\sqrt[n]{a}$

The radical expression

$\sqrt[n]{a}$ reads as "n^{th} root of a".

n is called the *index* and represents the numbers 2, 3, 4, so on.
A real number a is called *radicand* of the expression.

$$\sqrt[n]{a} = b \text{ means } b^n = a.$$

If n is an even number *ie:* 2,4,6…, then a must be nonnegative.

Example 9: Evaluate each expression:

59) $\sqrt[4]{-16}$ is *undefi*

ned because index 4 is even, and radicand -16 is negative.

65) $\sqrt[5]{-\frac{1}{32}}$ is *defined* because index 5 is odd, therefore radicand can be any real number.

Calculating the n^{th} root $\sqrt[n]{}$

If n is *odd*, then $\sqrt[n]{a^n} = a$,

If n is *even*, then $\sqrt[n]{a^n} = |a|$

Example 10:

$\sqrt[5]{(-3)^5} = -3$, $\sqrt[4]{(-3)^4} = |-3| = 3$, $\sqrt[3]{(-2)^3} = -2$, $\sqrt[4]{(-2)^4} = |-2| = 2$

$\sqrt[5]{3^5} = 3$, $\sqrt[4]{7^4} = 7$, $\sqrt[3]{2^3} = 2$, $\sqrt[4]{2^4} = 2$

Notice that to calculate expressions like $\sqrt[4]{7}$, we need calculator.

The Product and Quotient Rules for n^{th} Roots:

1) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ and $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

2) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$, and $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\quad b \neq 0$

Example 11: $\sqrt[4]{80} = \sqrt[4]{16 \cdot 5} = \sqrt[4]{16}\sqrt[4]{5} = \sqrt[4]{2^4}\sqrt[4]{5} = 2\sqrt[4]{5}$

$$\sqrt[5]{x^9} = \sqrt[5]{x^5 \cdot x^4} = \sqrt[5]{x^5}\,\sqrt[5]{x^4} = x\sqrt[5]{x^4}$$

Example 12: Simplify:

1) $\sqrt[4]{48x^9} =$

2) $\sqrt[4]{16x^9} =$

3) $\sqrt[3]{9x^2} \cdot \sqrt[3]{3x} =$

4) $\sqrt[3]{27x^5} =$

5) $\dfrac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}} =$

6) $\dfrac{\sqrt[4]{162x^5}}{\sqrt[4]{2x}} =$

Definition of $a^{\frac{1}{n}}$ & $a^{\frac{m}{n}}$

If $\sqrt[n]{a}$ is a real number, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}\text{ , and}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a}^{\,m}$$

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{\sqrt[n]{a^m}}, a \neq 0$$

Example 13: Evaluate each expression without using a calculator.

1) $125^{\frac{2}{3}} = \sqrt[3]{125}^{\,2}$
$= \sqrt[3]{5^3}^{\,2}$
$= 5^2$
$= 25$

2) $27^{\frac{4}{3}} =$

3) $32^{-\frac{4}{5}} =$

Example 14: Simplify using properties of exponents:

1) $\left(7\,x^{\frac{1}{3}}\right)\left(2\,x^{\frac{1}{4}}\right) =$

2) $\dfrac{20x^{\frac{1}{2}}}{5x^{\frac{1}{4}}} =$

3) $\left(25x^4y^6\right)^{\frac{1}{2}} =$

4) $\dfrac{\left(3y^{\frac{1}{4}}\right)^3}{y^{\frac{1}{12}}} =$

Example 15: Simplify by reducing the index of the radical

1) $\sqrt[9]{x^6y^3} =$

2) $\sqrt[4]{x^{10}y^6}$

3) $\sqrt[12]{x^4y^8} =$

4) $\sqrt[6]{x^{10}y^8}$

5) $\sqrt[15]{x^5y^{10}} =$

6) $\sqrt[14]{x^{28}y^7}$

P.4: Polynomials:

Definition of a polynomial in x:

A polynomial in x is an algebraic expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \dots + a_1x^1 + a_0$$

Where $a_n, a_{n-1}, \dots, a_0$ are real numbers, $a_n \neq 0$, and n is a nonnegative integer. The polynomial is of **degree *n*** , a_n is **leading coefficient**, and a_0 is the **constant term**.

The **degree of polynomial** is the greatest degree of all the terms.

A polynomial that has exactly **one term** is called a **monomial**.

A polynomial that has **two terms** is called a **binomial**.

A polynomial that has **three terms** is called a **trinomial.**

Example 1: Is the algebraic expression a polynomial? If it is, write the polynomial in standard form and find the degree of polynomial:

A) $x^2 - 8x^3 + 15x^4 + 91$

B) $2x + 3x^{-1} - 5$

C) $\dfrac{2x+3}{x}$

D) $\sqrt{3}\,x^3 + 6 - \dfrac{2}{3}x^4 - 4x - 5x^2$

E) $x^{\frac{2}{3}} + 2x^2 - 1$

F) $3x^4 - 7x + \dfrac{2}{x}$

P.4: Polynomials:

Adding and Subtracting Polynomials:

Polynomials are added and subtracted by combining like terms

Example 2: Perform the indicated operations write the resulting polynomial in standard form and indicate its degree

$A)\ (5x^3 + 3x^2 - x + 5) + (-2x^3 - 2x^2 + 4x - 2) =$

$B)\ (18x^4 - 2x^3 + 8 - 7x) - (6x^3 + 9x^4 - 5x + 7) =$

$C)\ (3x^3 + x^2 - x + 1) - (x^3 - 2x^2 + x - 4)$

$D)\ (8x^2 + 7x - 5) - (3x^2 - 4x) - (-6x^3 - 5x^2 + 3) =$

P.4: Polynomials:

Multiplying Polynomials:

Multiply each term of one polynomial by each term of the other polynomial. Then Combine like terms.

Example 3: Find each product

$A)\ (5x-2)(3x^2-5x+4)$

$B)\ (8x^3+3)(x^2-4x)$

Special Products:

$(A-B)(A+B)=A^2-B^2$	35) $(5-7x)(5+7x)=$
$(A+B)^2=A^2+2AB+B^2$	43) $(2x+3)^2=$
$(A-B)^2=A^2-2AB+B^2$	47) $\left(4x^2-1\right)^2=$
$(A+B)^3=A^3+3A^2B+3AB^2+B^3$	53) $(x+2)^3=$
$(A-B)^3=A^3-3A^2B+3AB^2-B^3$	57) $(3x-4)^3=$

P.4: Polynomials:

Polynomial in Two Variables:

Polynomial in two variables x and y contains the sum of one or more monomials in the form $\boldsymbol{ax^n\ y^m}$. $\boldsymbol{a}$ is the coefficient. The exponents, $\boldsymbol{m}$ and $\boldsymbol{n}$ represent whole numbers.

The degree of the monomial $\boldsymbol{ax^n\ y^m}$ is $\boldsymbol{m+n}$.
The degree of Polynomial in two variables: is the highest degree of all its terms.

Example 4: Is the algebraic expression a polynomial? If it is, find the degree of polynomial:

$a)\ 3x^2y^3 + 2x^2y + 1$

$b)\ 3x^2y^{-1} + 3x^2y$

$c)\ 2x^2y + 3x^4y^{\frac{1}{2}} - 5$

$d)\ 3x^5y^3 + 2x^9y + 1$

Example 5: Perform the indicated operations. Indicate the degree of the resulting polynomials.

$A)\quad (7x^4y^2 - 5x^2y^2 + 3xy) + (-18x^4y^2 - 6x^2y^2 - xy)$

$B)\quad (5x^4y^2 + 6x^3y - 7y) - (3x^4y^2 - 5x^3y - 6y + 8x)$

P.4: Polynomials:

Example 6: Find the product:

$A)\ (x+5y)(7x+3y)$

$B)\ (5x+3y)^2$

$C)\ (x-y)(x^2+xy+y^2)$

$D)\ (3xy-1)(5xy+2)$

$E)\ (x^2y^2-3)^2$

$F)\ (7xy^2-10y)(7xy^2+10y)$

P5: Factoring Polynomials:

Factoring a polynomial expressed as the sum of monomials means finding an equivalent expression that is a product.

In this section, we will be *factoring over the set of integers*, meaning that the coefficients in the factors are integers. Polynomials that cannot be factored using integer coefficients are called *irreducible over the integers*, or *prime*.

We will discuss basic techniques for factoring polynomials:

1) Common factor:

Greatest common factor (GCF): is an expression of the highest degree that divides each term of the polynomial.

Example 1: Factor:

$A)$ $4x^2 - 8x$

$\boldsymbol{B})$ $x^2(x-3) + 12(x-3)$

$B)$ $18x^3 - 27x^2$

$\boldsymbol{D})$ $2x(x+1) + 3(x+1)$

2)Factoring by grouping:

Some polynomials have only the greatest common factor of 1. However, by a suitable grouping of the terms, it still may be possible to factor. This process, called factoring by grouping.

Example2: Factor:

$a)$ $x^3 + 6x^2 - 2x - 12$

$b)$ $3x^3 - 2x^2 - 6x + 4$

P5: Factoring Polynomials:

3)Factoring trinomial $ax^2 + bx + c$

To factor a trinomial of the form $ax^2 + bx + c$, a little trial and error may be necessary.

$ks = c$

$$(nx + k)(mx + s) = ax^2 + bx + c$$

$nm = a$

1. Find two numbers n, m whose product is a and two numbers k, s whose product is c.

2. By trial and error, we must get $ns + km = b$

ns

$$(nx + k)(mx + s) = ax^2 + bx + c$$

km

3. If no such combination exists, then trinomial is prime.

Example 3: Factor the following trinomials.

19) $x^2 - 2x - 15 =$

20) $x^2 + 5x - 14 =$

25) $3x^2 - 25x - 28 =$

29) $4x^2 + 16x + 15 =$

33) $20x^2 + 27x - 8 =$

37) $6x^2 - 5xy - 6y^2 =$

51) $x^2 - 14x + 49 =$

55) $9x^2 - 6x + 1 =$

4) Factoring difference of two square:

$$A^2 - B^2 = (A - B)(A + B)$$

Remark: $A^2 + B^2$ is prime (factored completely)

Example4: Factor completely:

1) $x^2 - 4 =$	2) $x^4 - 16 =$	3) $64x^2 - 81 =$

4) $16x^4 - 81 =$	5) $x^2 + 36 =$

5) Factoring the sum or Difference of two cubes:

$$A^3 + B^3 = (A + B)\left(A^2 - AB + B^2\right)$$

$$A^3 - B^3 = (A - B)\left(A^2 + AB + B^2\right)$$

Example 5: Factor completely:

59) $x^3 - 64 =$	63) $64x^3 + 27 =$
64) $8x^3 + 27 =$	65) $27x^3 - 64 =$

P5: Factoring Polynomials:

Example 6: Factor completely or state that the polynomial is prime:

1) $y^5 - 81y =$	2) $x^3 + 2x^2 - 9x - 18 =$
3) $x^2 + 4 =$	4) $x^3 - 4x =$
5) $x^4 + 2x^3 - 15x^2 =$	6) $9x^2 - 16 =$
7) $8x^3 + 27 =$	8) $18x^3 - 9x^2 - 8x + 4 =$
9) $x^4 + 3x^3 - 10x^2 =$	10) $x^2 + 2x - 15 =$

Rational Expressions

A rational expression is the quotient of two polynomials. Some examples are

$$\frac{x-2}{4}, \qquad \frac{4}{x-2}, \qquad \frac{x}{x^2-1}, \qquad \frac{x^2+1}{x^2+2x-3}$$

The set of real numbers for which an algebraic expression is defined is the **domain** of the expression. Because rational expressions indicate division and division by zero is undefined, **we must exclude numbers from a rational expression's domain that make the denominator zero.**

Example1: Find the numbers that must be ***excluded*** from the domain of each rational expression, then ***write the domain***:

$A)\ \dfrac{13}{x+9}$

$B)\ \dfrac{x+5}{x^2-25}$

$C)\ \dfrac{x-1}{x^2+11x+10}$

$D)\ \dfrac{x-1}{x^2+1}$

$E)\ \dfrac{x-2}{x^2-5x+6}$

$F)\ \dfrac{x-4}{x^2-16}$

P6: Rational Expressions:

1-Simplifying Rational Expressions

A rational expression is simplified if its numerator and denominator have no common factor than 1 or -1. The following procedure can be used to simplify rational expressions.

1. Factor the numerator and the denominator completely.
2. Divide both the numerator and the denominator by any common factors.

Example2: Simplify each rational expression. Find all numbers that must be excluded from the domain of the ***simplified rational expression***.

$A)\ \dfrac{x^2-12x+36}{4x-24}$ $\qquad$ $B)\ \dfrac{y^2+7y-18}{y^2-3y+2}$

$C)\ \dfrac{x^2+6x+9}{2x+6}$ $\qquad$ $D)\ \dfrac{2x^2-x}{3x^2-2x}$

$E)\ \dfrac{y^2+7y-18}{y^2-3y+2}$ $\qquad$ $F)\ \dfrac{x^2-9}{x^2-5x+6}$

2-Multiplying Rational Expressions:

1. Factor all numerators and denominators completely.
2. Divide numerators and denominators by common factors.
3. Multiply the remaining factors in the numerators and multiply the remaining factors in the denominators.

Example3: *Multiply*

$A)\ \dfrac{6x+9}{3x-15}\cdot\dfrac{x-5}{4x+6}$

$B)\ \dfrac{x^2+5x+6}{x^2+x-6}\cdot\dfrac{x^2-9}{x^2-x-6}$

$C)\ \dfrac{x^3-8}{x^2-4}\cdot\dfrac{x+2}{3x}$

$D)\ \dfrac{x^2-4}{x-2}\cdot\dfrac{x+1}{3x+6}$

3. Dividing Rational Expressions

The quotient of two rational expressions is the product of the first expression and the multiplicative inverse, or reciprocal of the second expression. The reciprocal is found by interchanging the numerator and the denominator. Thus, we find the quotient of two rational expressions by inverting the divisor and multiplying.

Example 4: *Divide:*

$A)\ \dfrac{x^2-4}{x}\div\dfrac{x+2}{x-2}$

$B)\ \dfrac{x^2-25}{2x-2}\div\dfrac{x^2+10x+25}{x^2+4x-5}$

P6: Rational Expressions:

Adding and Subtracting Rational Expressions with the Same Denominator

We add or subtract rational expressions with the same denominator by
(1) adding or subtracting the numerators,
(2) placing this result over the common denominator, and
(3) simplifying, if possible.

Example 5: *Add and Subtract the following expressions.*

$A)\ \dfrac{2x}{x+2}+\dfrac{4}{x+2}$

$B)\ \dfrac{3x}{x-3}-\dfrac{2-x}{x-3}$

$C)\ \dfrac{x^2-2x}{x^2+3x}+\dfrac{x^2+x}{x^2+3x}$

$D)\ \dfrac{x^2+3x}{x^2+x-12}-\dfrac{x^2-12}{x^2+x-12}$

$D)\ \dfrac{x^2-6x}{x^2+2x}+\dfrac{x^2+5x}{x^2+2x}$

$E)\ \dfrac{x^2+4x}{x^2-7x-8}-\dfrac{x^2-4}{x^2-7x-8}$

P6: Rational Expressions:

The least common denominator, or LCD, of several rational expressions is a polynomial consisting of the product of all prime factors in the denominators, with each factor raised to the greatest power of its occurrence in any denominator. When adding and subtracting rational expressions that have different denominators with one or more common factors in the denominators, it is efficient to find the least common denominator first.

Finding the Least Common Denominator

1. Factor each denominator completely.
2. List the factors of the first denominator.
3. Add to the list in step 2 any factors of the second denominator that do not appear in the list.
4. Form the product of the factors from the list in step 3. product is the least common denominator.

Adding and Subtracting Rational Expressions That Have Different Denominators

1. Find the LCD of the rational expressions.

2. Rewrite each rational expression as an equivalent expression whose denominator is the LCD. To do so, multiply the numerator and the denominator of each rational expression by any factor(s) needed to convert the denominator into the LCD.

3. Add or subtract numerators, placing the resulting expression over the LCD.

4. If possible, simplify the resulting rational expression.

Example 6: *Add or subtract.*

$A)\ \dfrac{8}{x-2}+\dfrac{2}{x-3}$

$B)\ \dfrac{2x}{x+2}+\dfrac{x+2}{x-2}$

P6: Rational Expressions:

$C)\ \dfrac{5}{2x+8}+\dfrac{7}{3x+12}$

$D)\ \dfrac{x+3}{x^2-1}-\dfrac{x+2}{x-1}$

$E)\ \dfrac{3x}{x^2+3x-10}-\dfrac{2x}{x^2+x-6}$

$F)\ \dfrac{4x-1}{2x^2+5x-3}-\dfrac{x+3}{6x^2+x-2}$

REVIEW EXERCISES

P.1

Evaluate each algebraic expression for the given value or values of the variables.

1. $3+6(x-2)^3$ for $x=4$

2. $x^2-5(x-y)$ for $x=6$ and $y=2$

3. $x^2+5y-2xy^2$ for $x=5, y=4$

4. $\dfrac{3x^3-2y}{x+y^2}$ for $x=3, y=-2$

5. $\dfrac{2x^2y}{x-y}$ for $x=2, y=-3$

6. $\dfrac{5x+y}{xy-5x}$ for $x=2, y=-3$

7. *Rewrite the expression without absolute value bars*

$A)$ $|12-|-15||$

$B)$ $|-|-3|-|-25||$

8. *Simplify the given algebraic expression.*

$A)$ $2(6y-4)-3[2(3x-y)-x+y]$

$B)$ $8-3[5-2(4x-7)]$

$C)$ $5(2x-3)+7x$

$D)$ $\frac{1}{7}(7x)-[(4x)-(-2y)]-(-3x)$

$E)$ $3(4y-5)-(7y+2)$

$F)$ $8-2[3-(5x-1)]$

P.2

28) $\left(-2x^4y^3\right)^3$

29) $\left(-5x^3y^2\right)\left(-2x^{-11}y^{-2}\right)$

31) $\dfrac{7x^5y^6}{28x^{15}y^{-2}}$

P.3

Use the product rule $\left(\sqrt{ab}=\sqrt{a}\sqrt{b}\right)$*, to simplify the expressions, assume that variables represent nonnegative real numbers*

$A)\ \sqrt{300}$

$B)\ \sqrt{12x^2}$

$C)\ \sqrt{10x}\sqrt{2x}$

$D)\ \sqrt{3x^2}\sqrt{15x}$

$E)\ \sqrt{27x^3}$

$F)\ \sqrt{15x}\sqrt{12x^3}$

P6: Rational Expressions:

Use the quotient rule $\left(\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}\right)$, *to simplify the expressions*

$A)\ \sqrt{\frac{121}{4}}$

$B)\ \frac{\sqrt{96x^3}}{\sqrt{2x}}$

$C)\ \frac{\sqrt{125x}}{\sqrt{5x^{-1}}}$

Add or subtract terms whenever possible.

$A)\ 2\sqrt{50} + 3\sqrt{8}$

$B)\ 4\sqrt{72} - 2\sqrt{48}$

$C)\ 3\sqrt{48x} - 4\sqrt{27x}$

Rationalize the denominator

$A)\ \frac{\sqrt{2}}{\sqrt{3}}$

$B)\ \frac{5}{6+\sqrt{3}}$

$C)\ \frac{14}{\sqrt{7}-\sqrt{5}}$

P6: Rational Expressions:

Evaluate each expression or indicate that the root is not a real number.

A) $\sqrt[3]{125}$ | B) $\sqrt[5]{-32}$ | C) $\sqrt[4]{-125}$ | D) $\sqrt[4]{(-5)^4}$

Simplify the radical expressions

A) $\sqrt[3]{81}$ | B) $\sqrt[3]{y^5}$ | C) $\sqrt[4]{8}\sqrt[4]{10}$ | D) $4\sqrt[3]{16}+5\sqrt[3]{2}$ | E) $\dfrac{\sqrt[4]{32x^5}}{\sqrt[4]{16x}}$

Evaluate each expression

A) $16^{1/2}$ | B) $25^{-1/2}$ | C) $125^{1/3}$ | D) $27^{-1/3}$ | E) $64^{2/3}$ | F) $27^{-4/3}$

Simplify using properties of exponents.

A) $\left(5x^{\frac{2}{3}}\right)\left(4x^{\frac{1}{4}}\right)$ | B) $\left(12x^{-\frac{2}{5}}\right)\left(\frac{2}{3}x^{\frac{1}{3}}\right)$ | C) $\left(\frac{3}{4}x^{\frac{4}{3}}\right)\left(\frac{4}{15}x^{-\frac{1}{4}}\right)$

P6: Rational Expressions:

P.4

Perform the indicated operations. Write the resulting polynomial in standard form and indicate its degree.

$A)\ (-6x^3+7x^2-9x+3)+(14x^3+3x^2-11x-7)$

$B)\ (13x^4-8x^3+2x^2)-(5x^4-3x^3+2x^2-6)$

$C)\ (7x^2-8xy+y^2)+(-8x^2-9xy-4y^2)$

$D)\ (13x^3y^2-5x^2y-9x^2)-(-11x^3y^2-6x^2y+3x^2-4)$

Find each product

$A)\ (3x-2)(4x^2+3x-5)$

$B)\ (3x-5)(2x+1)$

$C)\ (4x+5)(4x-5)$

$D)\ (2x+5)^2$

$E)\ (3x-4)^2$

$F)\ (2x+1)^3$

P6: Rational Expressions:

$H)$ $(5x-2)^3$

$I)$ $(x+7y)(3x-5y)$

$J)$ $(3x-5y)^2$

$K)$ $(3x^2+2y)^2$

$L)$ $(7x+4y)(7x-4y)$

P.5

Factor completely, or state that the polynomial is prime.

1) $15x^3+3x^2$

2) $x^2-11x+28$

3) $15x^2-x-2$

4) $64-x^2$

5) x^2+16

6) $3x^4-9x^3-30x^2$

7) $20x^7-36x^3$

8) $x^3-3x^2-9x+27$

P6: Rational Expressions:

9) $16x^2 - 40x + 25$

10) $x^4 - 16$

11) $y^3 - 8$

12) $x^3 + 64$

13) $3x^4 - 12x^2$

14) $27x^3 - 125$

P.6

Simplify each rational expression. Also, list all numbers that must be excluded from the domain.

A) $\dfrac{x^3 + 2x^2}{x + 2}$

B) $\dfrac{x^2 + 2x}{x^2 + 4x + 4}$

C) $\dfrac{x^2 + 3x - 18}{x^2 - 36}$

Multiply or divide as indicated

A) $\dfrac{x^2 + 6x + 9}{x^2 - 4} \cdot \dfrac{x + 3}{x - 2}$

B) $\dfrac{6x + 2}{x^2 - 1} \div \dfrac{3x^2 + x}{x - 1}$

P6: Rational Expressions:

Add or subtract as indicated

$A)\ \dfrac{2x-7}{x^2-9}-\dfrac{x-10}{x^2-9}$

$B)\ \dfrac{3x}{x+2}+\dfrac{x}{x-2}$

$C)\ \dfrac{x}{x^2-9}+\dfrac{x-1}{x^2-5x+6}$

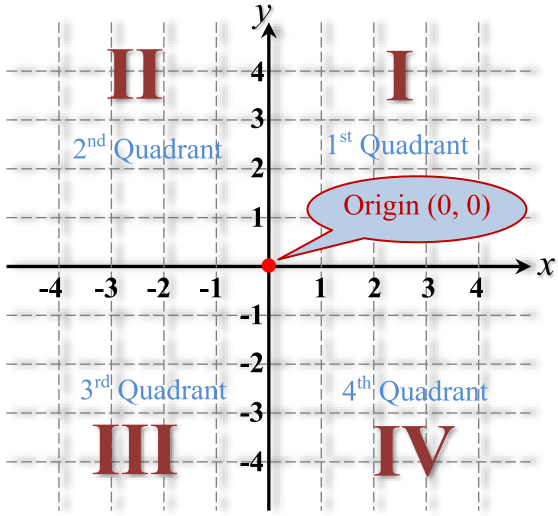
$D)\ \dfrac{4x-1}{2x^2+5x-3}-\dfrac{x+3}{6x^2+x-2}$

Chapter1: Equations and Inequalities.

1.1: Graphs:

Rectangular Coordinate System:

- The horizontal number line is the ***x-axis***.
- The vertical number line is the ***y-axis***.
- The point of intersection of these axes is their zero points, called the ***origin***.
- The axes divide the plane into four quarters, called ***quadrants***.
- Each point in the rectangular coordinate system corresponds to an ***ordered pair*** of real numbers, $(\boldsymbol{x}, \boldsymbol{y})$.



❖ Plot the Point:

Example 1: Plot the given points in rectangular coordinate system.

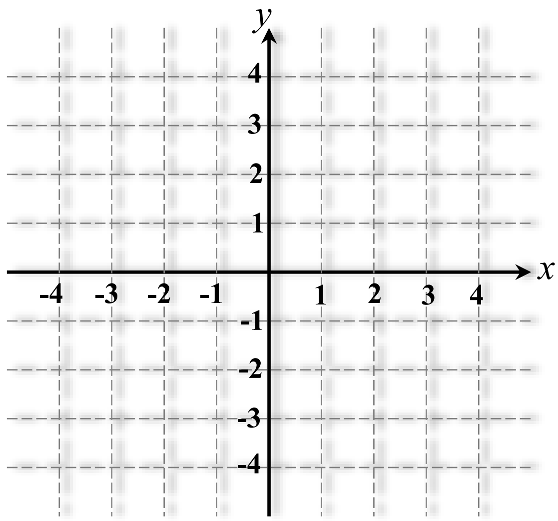
$A)\ (-2, 3)$

$B)\ (-3, -2)$

$C)\ (-4, 0)$

$D)\ (4, 3)$

$E)\ (0, 3)$



Chapter1: Equations and Inequalities.

1.1: Graphs:

❖ Graph of Equations:

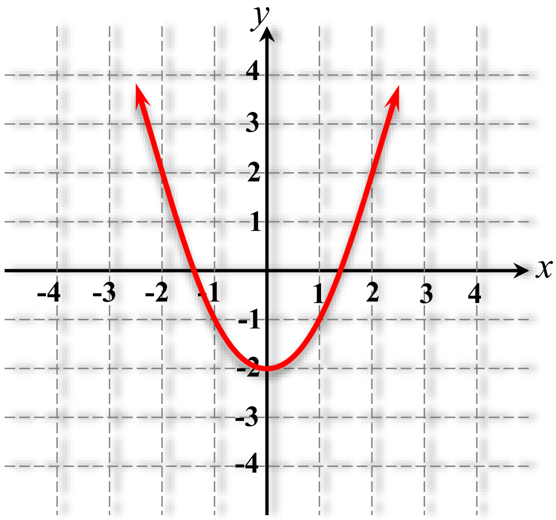
A relationship between two quantities can be expressed as an equation in two variables. such as: $y = x^2 - 2$

The graph of an equation in two variables is the set of all points whose coordinates satisfy the equation.

The graph of $y = x^2 - 2$

Let $x = 2$, then
$y = 2^2 - 2 = 4 - 2 = 2$
The point $(2,2)$ satisfies the equation.

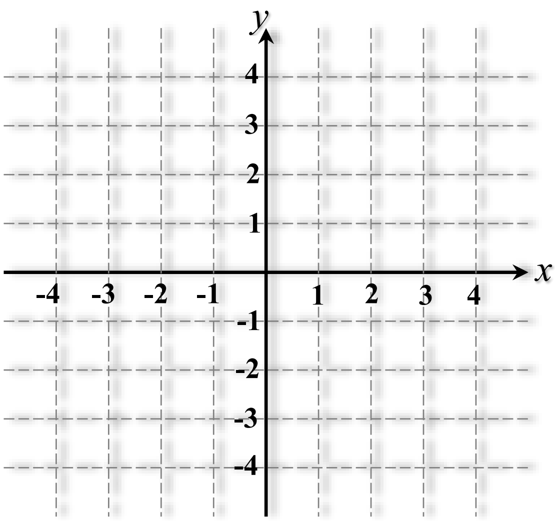
Let $x = -2$,
then $y = (-2)^2 - 2 = 4 - 2 = 2$
The point $(-2,2)$ satisfies the equation.



Example 2: Graph each equation. Let $x = -2, -1, 0, 1, 2$

$A)\ y = 2x + 1$

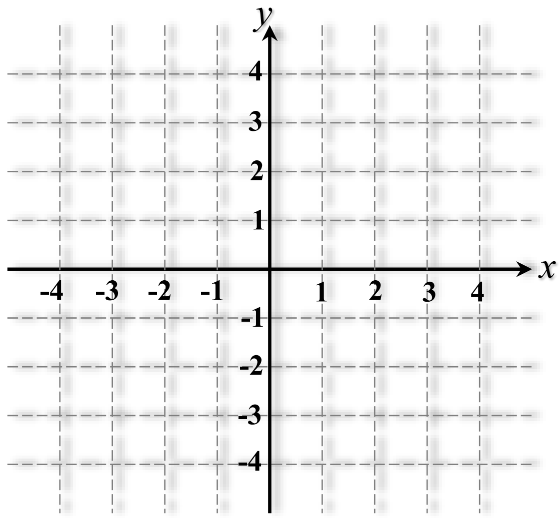
x	$y = 2x + 1$	(x, y)



1.1: Graphs:

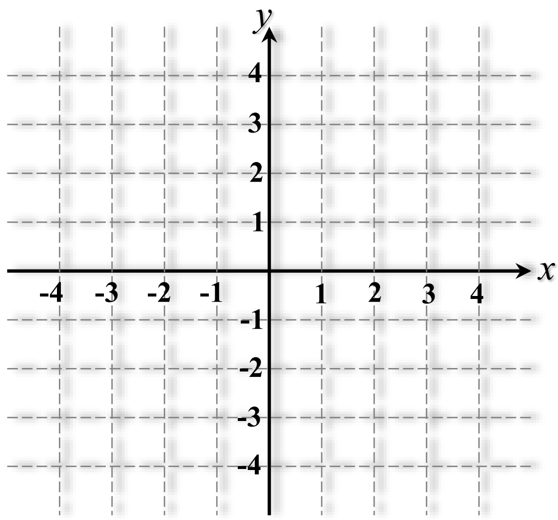
$B)\ y = |x| + 1$

x	$y = \lvert x \rvert + 1$	(x, y)



$C)\ y = 4 - x^2$

x	$y = 4 - x^2$	(x, y)

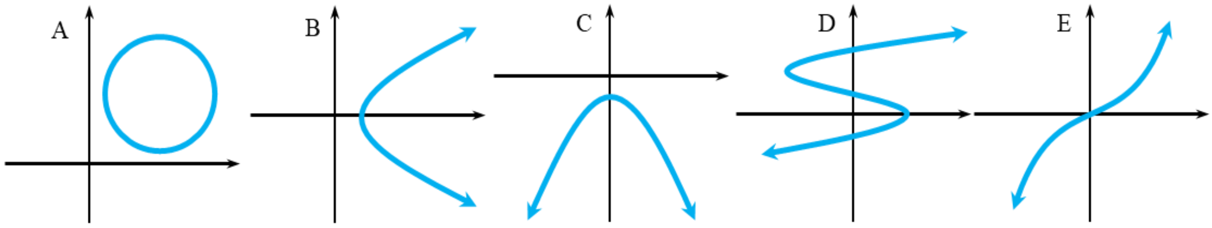


Chapter1: Equations and Inequalities.

1.1: Graphs:

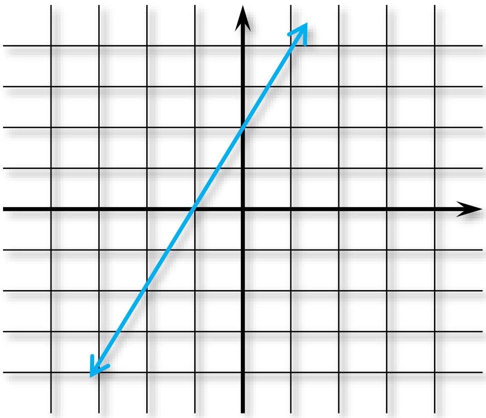
❖ Intercepts:

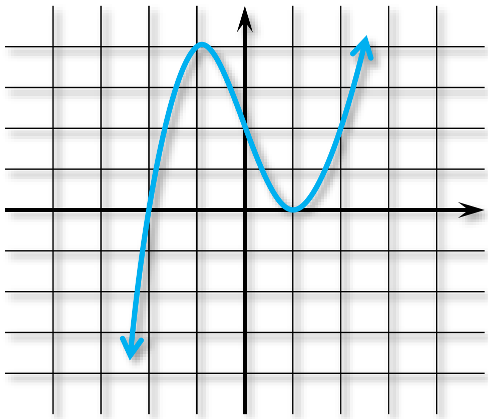
x-intercept of a graph is the x-coordinate of a point where the graph intersects the x-axis.The y-coordinate corresponding to an x-intercept is always zero $(x, 0)$.
y-intercept of a graph is the y-coordinate of a point where the graph intersects the y-axis. The x-coordinate corresponding to an y-intercept is always zero $(0, y)$.

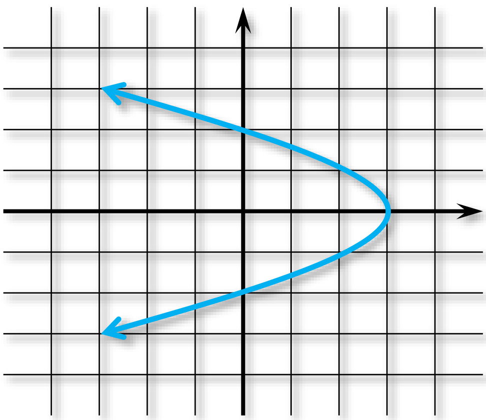


No intercepts | One x-int.,no y-int. | One y-int.,no x-int. | One x-int.,three y-int. | One x- int.,one y-int

Use the graphs to find the x-intercepts and y-intercepts.







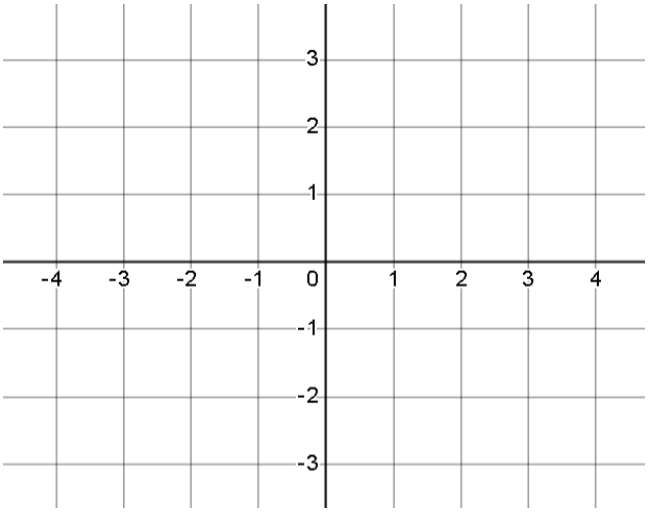
1.1: Graphs:

Example 4:

i) For the equation: $y = x + 1$

a) Graph the equation for Let $x = -1, 0 \ and \ 1$.

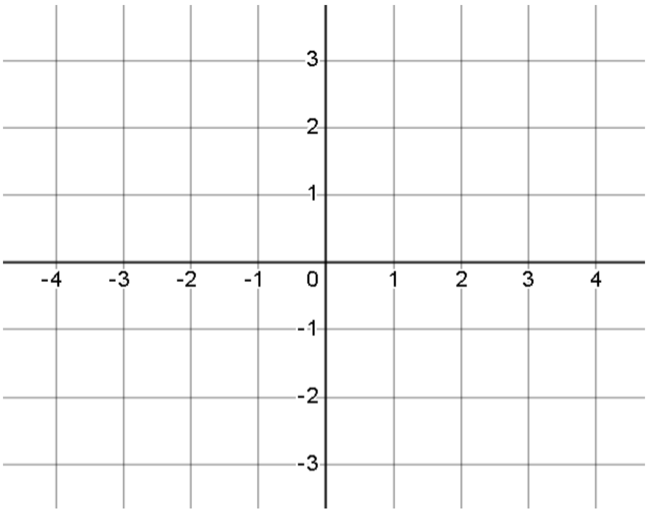
b) Use the graph to determine x-intercepts and y-intercepts



ii) For the equation: $y = x^2 - 4$

a) Graph the equation for Let $x = -2, 0 \ and \ 2$.

b) Use the graph to determine x-intercepts and y-intercepts



Chapter1: Equations and Inequalities.

1.1: Graphs:

Chapter1: Equations and Inequalities.

1.2: Linear Equations and Rational Equations:

❖ Linear Equations

Definition of a Linear Equation

A linear equation in one variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real numbers and $a \neq 0$.

Solving a Linear Equation

1. Simplify the algebraic expression on each side by removing grouping symbols and combining like terms.
2. Collect all the variable terms on one side and all the numbers, or constant terms, on the other side.
3. Isolate the variable and solve.
4. Check the proposed solution in the original equation.

Example 1: Solve each linear equation.

$A)\ 3x + 4 = 0$

$B)\ 3(x-4) - 4(x-3) = x + 3 - (x-2)$

$C)\ 16 = 3(x-1) - (x-7)$

$D)\ 8(x+5) = 2x - 2$

1.2: Linear Equations and Rational Equations:

❖ Linear Equations with Fractions

Example 2: Solve each equation.

$A)\quad \frac{3x}{5} - x = \frac{x}{10} - \frac{5}{2}$

$B)\quad \frac{x+3}{6} = \frac{3}{8} + \frac{x-5}{4}$

1.2: Linear Equations and Rational Equations:

❖ Rational Equations:

A **rational equation** is an equation containing one or more rational expressions.
To solve a rational equation:

1) Find the excluded value.
2) Find L.C.D
3) Multiply both sides of the equation by L.C.D
4) Solve

Example 3: Solve each equation:

$A)\quad \frac{2}{x}+3=\frac{5}{2x}+\frac{13}{4}$

$B)\quad \frac{8x}{x+1}=4-\frac{8}{x+1}$

Chapter1: Equations and Inequalities.

1.2: Linear Equations and Rational Equations:

$C)\ \dfrac{1}{x-4} - \dfrac{5}{x+2} = \dfrac{6}{x^2-2x-8}$

$D)\ \dfrac{2}{x+1} - \dfrac{1}{x-1} = \dfrac{2x}{x^2-1}$

$E)\ \dfrac{3}{x+2} + \dfrac{2}{x-2} = \dfrac{8}{(x+2)(x-2)}$

1.2: Linear Equations and Rational Equations:

4- Recognize identities, conditional equations, and inconsistent equations:

Equations can be placed into categories that depend on their solution sets.

1- An equation that is true for all real numbers for which both sides are defined is called an **identity**.

2-An equation that is not an identity, but that is true for at least one real number, is called a **conditional equation**

3-An **inconsistent equation** is an equation that is not true for even one real number.

Example 4: Solve each equation. Then determine whether the equation is an **identity, a conditional equation,** or an **inconsistent** equation

$\boldsymbol{A)}\ \ \mathbf{10x + 3 = 8x + 3}$

$\boldsymbol{B)}\ \mathbf{3(x + 2) = 7 + 3x}$

$\boldsymbol{C)}\ \mathbf{5x + 9 = 9(x + 1) - 4x}$

Chapter1: Equations and Inequalities.

1.2: Linear Equations and Rational Equations:

D) $\frac{2}{x+1} - \frac{1}{x-1} = \frac{2x}{x^2-1}$

$E)$ $\frac{1}{x-4} - \frac{5}{x+2} = \frac{6}{x^2-2x-8}$

1.2: Linear Equations and Rational Equations:

Introduction to Sustainability:

Sustainability means meeting the needs of the present without compromising the ability of future generations to meet their own needs. It focuses on balancing environmental care, social well-being, and economic growth to ensure a healthy planet and a better quality of life for everyone.

Sustainable Development Goals (SDGs):

Here is the image showing the 17 global goals adopted by the United Nations

1.2: Linear Equations and Rational Equations:

Example 4:

Goal 13: Climate Action

Take urgent action to combat climate change and its impacts.

$$x + y = 100$$

Linear Equation with Two Variables, where:

- x: Amount of carbon emissions reduced (in tons) by Country A.
- y: Amount of carbon emissions reduced (in tons) by Country B.
- The target is to achieve a total reduction of 100 tons of carbon emissions.

1. If Country A reduces 40 tons of carbon emissions, how much must Country B reduce to meet the goal?
2. What values of x and y satisfy the equation if both countries reduce the same amount?
3. If Country B cannot reduce more than 60 tons, what is the minimum reduction required by Country A?

1.2: Linear Equations and Rational Equations:

Example 5:

Goal 4: Quality Education

Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all.

$$x + 2y = 120$$

Linear Equation with Two Variables, where:

x: Number of students provided with educational tablets.

y: Number of teacher training workshops conducted.

The target is to reach 120 educational development units, where each teacher training workshop is counted as two units due to its deeper impact on education quality.

1) If 20 training workshops are conducted, how many tablets need to be distributed to meet the goal?

2) If only 60 tablets can be provided, how many workshops must be held?

3) Do the values $x = 40,\ y = 40$ satisfy the equation? Why or why not?

Practice: 3,9,29,43

1.4: Complex Numbers

The Imaginary Unit i

The imaginary unit $\boldsymbol{i}$ is defined as

$$i = \sqrt{-1}\text{, where } i^2 = -1$$

Example1: $\sqrt{-a} = \sqrt{a}\, i$

$\sqrt{-25} =$ $\quad$ $\sqrt{-16} =$ $\quad$ $\sqrt{-9} =$ $\quad$ $\sqrt{-5} =$ $\quad$ $\sqrt{-3} =$

$$(ai)^2 = (-ai)^2 = -a^2$$

$(-i)^2 =$ $\quad$ $(2i)^2 =$ $\quad$ $(-2i)^2 =$ $\quad$ $(3i)^2 =$ $\quad$ $(-3i)^2 =$

Complex Numbers and Imaginary Numbers

The set of all numbers in the form

$$a + bi$$

with real numbers a and b, and i, the imaginary unit, is called the set of ***complex numbers***. "The real number $\boldsymbol{a}$ is called the ***real part*** and the real number $\boldsymbol{b}$ is called the ***imaginary part*** of the complex number $\boldsymbol{a + bi}$**.**

Example2: The complex number (imaginary number) $3 - 4i$ has
3 as the *real part* and **-4 as the *imaginary part.***

The complex number $5i$ is pure imaginary number which is the same as $0 + 5i$
0 is the *real part* and **5 is the *imaginary part.***

-7 is the same as $-7 + 0i$. **-7 is the real part, 0 is the imaginary part.**

1.4: Complex Numbers

Conjugate of a Complex Number

The ***complex conjugate*** of the number $a + bi$ is $a - bi$, and

the ***complex conjugate*** of the number $a - bi$ is $a + bi$

Example3:

The complex conjugate of $3 - 5i$ is $3 + 5i$.

The complex conjugate of $7 + 2i$ is $7 - 2i$.

The complex conjugate of $8i$ is $-8i$.

The complex conjugate of 4 is 4

Example4: Find the following:

Complex number	Real part	Imaginary	Conjugate
$a + bi$	a	b	$a - bi$
$a - bi$	a	$-b$	$a + bi$
$2 - 7i$			
$-5 + 9i$			
$-4i$			
$\sqrt{9} - \sqrt{-25}$			
$\sqrt{2}$			
$\frac{1}{4} - \frac{3}{4}i$			
$\sqrt{-9}$			

1.4: Complex Numbers

Adding and Subtracting Complex Numbers.

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

Example5: Find the following:

1) $(7+2i)+(1-4i)$	2) $(3+2i)-(5-7i)$
3) $6-(-5+4i)-(-13-i)$	4) $(5-2i)-(3-4i)$

Multiplying Complex Numbers:

$$(a+bi)(c+di)=ac+adi+bci+bdi^2$$

$$=ac-bd+(ad+bc)i$$

Multiplication of conjugates: $(a+bi)(a-bi)=a^2+b^2$

Example6: Find the following:

1) $(-5+4i)(3+i)$	2) $(3+5i)(3-5i)$
3) $(2+3i)^2=$	4) $(2i-3)^2=$
5) $(-2-i)^2=$	6) $(3i-4)^2=$

1.4: Complex Numbers

Dividing Complex Numbers:

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{(a+bi)(c-di)}{c^2+d^2}$$

Example7: Perform the indicated operations and write the result in standard form:

1) $\frac{2}{3-i} =$	2) $\frac{3i}{2+i} =$	
3) $5\sqrt{-16} + 3\sqrt{81}$	4) $\left(2-\sqrt{-9}\right)^2 =$	5) $\frac{2+3i}{2+i}$

1.4: Complex Numbers

6) $\sqrt{-8}\left(\sqrt{-9}-\sqrt{2}\right)$	7) $\sqrt{-9}\sqrt{-4}-\sqrt[3]{-27}$	8) $\frac{4i)}{10-5i}$
9) $\frac{2}{3-2i}=$	10) $\frac{3}{5-i}$	11) $2i(2-3i)^2=$
12) $\sqrt{-36}\left(\sqrt{-4}+\sqrt{-9}\right)$	13) $\sqrt{-25}\left(\sqrt{-16}-\sqrt{-8}\right)$	

1.5: Quadratic Equations:

Definition of a Quadratic Equation

A quadratic equation in x is an equation that can be written in the general form

$$ax^2 + bx + c = 0$$

where a, b. and c are real numbers. with $a \neq 0$. A quadratic equation in x is also called a second-degree polynomial equation in x.

Solving Quadratic Equations

➤ By Factoring:

The Zero-Product Principle
If the product of two algebraic expressions is zero, then at least one of the factors is equal to zero.

If $AB = 0$, then $A = 0$ or $B = 0$.

Example1: Solve each equation:

1) $(2x+1)(x-2) = 0$
2) $6x^2 + 11x - 10 = 0$
3) $3x^2 + 12x = 0$

1.5: Quadratic Equations:

4) $7 - 7x = (3x + 2)(x - 1)$

➢ Square Root Property:

If u is an algebraic expression and d is a nonzero real number, then $u^2 = d$ has exactly two solutions:

$$\text{If } u^2 = d, \text{ then } u = \sqrt{d} \quad \text{or} \quad u = -\sqrt{d}$$

If $x^2 = 5$, then $x = \sqrt{5}$, $x = -\sqrt{5}$

Example2: Solve:

1) $5x^2 + 1 = 51$

2) $2x^2 - 7 = -15$

3) $3(x - 4)^2 = 15$

4) $2(x - 1)^2 - 5 = 4$

5) $(3x + 2)^2 = 9$

6) $(x + 2)^2 - 4 = -3$

1.5: Quadratic Equations:

➢ Completing the Square:

To solve an equation in the form $ax^2 + bx + c = 0$ by completing the square:

1) Check if the coefficient of x^2 is one. $(a = 1)$
2) Move the constant term to the other side.

$$x^2 + bx = -c$$

3) Add $\left(\frac{b}{2}\right)^2$ to both sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

4) Factor and simplify.
5) Apply the square root property.

Example3: Solve by completing the square:

1) $x^2 - 6x - 11 = 0$

2) $x^2 + 8x + 4 = 0$

1.5: Quadratic Equations:

3) $x^2 - 4x - 1 = 0$

4) $x^2 - 6x + 7 = 0$

➢ The Quadratic Formula:

The solutions of a quadratic equation in general form $ax^2 + bx + c = 0$, with $a \neq 0$, are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is called Discriminant of the equation.

If $b^2 - 4ac > 0$, then the equation has two real solutions.

If $b^2 - 4ac = 0$, then the equation has one real solution (a repeated solution)

If $b^2 - 4ac < 0$, then the equation has two complex solutions, they are conjugate.

Example4: Solve using the quadratic formula:

1) $x^2 + 5x + 3 = 0$

2) $4x^2 = 2x + 7$

1.5: Quadratic Equations:

Example5: Compute the discriminant, then determine the number and type of solutions.

3) $2x^2 - 11x + 3 = 0$

4) $3x^2 - 8x + 7 = 0$

5) $9x^2 - 6x + 1 = 0$

6) $2x^2 - 11x + 3 = 0$

1.5: Quadratic Equations:

Example6: Solve by the method of your choice:

1) $5x^2 + 2 = 11x$

2) $(2x + 3)(x + 4) = 1$

3) $4x^2 - 16 = 0$

4) $2x^2 - 7x = 0$

1.5: Quadratic Equations:

5) $\dfrac{2x}{x-3}+\dfrac{6}{x+3}=-\dfrac{28}{x^2-9}$

6) $\dfrac{x}{x-2}+\dfrac{3}{x+2}=-\dfrac{4}{x^2-4}$

1.6: Other Types of Equations:

❖ Polynomial Equations:

To Solve a Polynomial Equation by Factoring:

1) Move all terms to one side and obtain zero on the other side.
2) Factor.
3) Apply the zero-product principle, setting each factor equal to zero.
4) Solve the equations in step 3.

Example1: Solve each equation:

1) $3x^3 + 2x^2 = 12x + 8$

2) $4y^3 + 8y^2 - y - 2 = 0$

3) $2x^4 = 16x$

4) $(2x + 1)(x - 2)(3x + 2) = 0$

1.6: Other Types of Equations:

❖ Radical Equations:

A radical equation is an equation in which the variable occurs in a square root, cube root, or any higher root.

Solving Radical Equations Containing nth Roots

1. If necessary. arrange terms so that one radical is isolated on one side of the equation.
2. Raise both sides of the equation to the nth power to eliminate the isolated nth root.
3. Solve the resulting equation. If this equation Still contains radicals. repeat steps 1 and 2.
4. Check all proposed solutions in the original equation.

Example2: Solve each equation:

1) $\sqrt{x+3} = x-3$

2) $\sqrt{5x+11} - 1 = x$

3) $\sqrt{-2x+30} = x-3$

4) $x - \sqrt{2x+5} = 5$

5) $\sqrt{x-1} = x-7$

6) $\sqrt{2x-2} = x-1$

1.6: Other Types of Equations:

❖ Equations with Rational Exponents:

Solving Radical Equations of the Form $x^{m/n} = k$

Assume that m and n are positive integers, $\frac{m}{n}$ is in lowest terms, and k is a real number.

1. Isolate the expression with the rational exponent.
2. Raise both sides of the equation to the $\frac{n}{m}$ power.

If m is even:	If m is odd:
$x^{m/n} = k$	$x^{m/n} = k$
$\left(x^{m/n}\right)^{n/m} = \pm k^{n/m}$	$\left(x^{m/n}\right)^{n/m} = k^{n/m}$
$x = \pm k^{n/m}$	$x = k^{n/m}$

It is incorrect to insert the ± symbol when the numerator of the exponent is odd. An odd index has only one root.

3. Check all proposed solutions in the original equation to find out if they are actual solutions or extraneous solutions.

Example3: Solve:

1) $x^{3/2} = 8$

2) $(x-4)^{3/2} = 27$

3) $2x^{5/2} - 64 = 0$

4) $(x-4)^{2/3} = 16$

1.6: Other Types of Equations:

❖ Equations that are quadratic in form:

Some equations that are not quadratic can be written as quadratic equations ***using substitution***.

Example4: Solve

1) $x^4 - 5x^2 + 4 = 0$

2) $9x^4 = 25x^2 - 16$

3) $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$

4) $x^{\frac{3}{2}} - 2x^{\frac{3}{4}} + 1 = 0$

1.6: Other Types of Equations:

❖ Equations Involving Absolute Value:

Rewriting an Absolute Value Equation without Absolute Value Bars
If ***c*** is a positive real number and ***u*** represents any algebraic expression, then

$|u| = c$ is equivalent to

$u = c$ or $u = -c$

Example5: Solve

1) $|x - 2| = 7$

2) $5|1 - 4x| - 8 = 7$

3) $7|5x| + 2 = 16$

4) $|x + 1| + 5 = 3$

5) $2|x - 6| + 3 = 3$

6) $5 + 2|1 + 4x| = 7$

1.6: Other Types of Equations:

$$|-2x+7| = 25$$

$\boxed{-2x+7=25}$

$$-2x+7=25$$
$$-2x+7-7=25-7$$
$$-2x=18$$
$$\frac{-2x}{-2}=\frac{18}{-2}$$
$$x=-9$$

$\boxed{-2x+7=-25}$

$$-2x+7=-25$$
$$-2x+7-7=-25-7$$
$$-2x=-32$$
$$\frac{-2x}{-2}=\frac{-32}{-2}$$
$$x=16$$

$$|-6x+3| = 27$$

$\boxed{-6x+3=27}$

$$-6x+3=27$$
$$-6x+3-3=27-3$$
$$-6x=24$$
$$\frac{-6x}{-6}=\frac{24}{-6}$$
$$x=-4$$

$\boxed{-6x+3=-27}$

$$-6x+3=-27$$
$$-6x+3-3=-27-3$$
$$-6x=-30$$
$$\frac{-6x}{-6}=\frac{-30}{-6}$$
$$x=5$$

$$|9-2x| = 3$$

$\boxed{9-2x=3}$

$$9-2x=3$$
$$9-9-2x=3-9$$
$$-2x=-6$$
$$\frac{-2x}{-2}=\frac{-6}{-2}$$
$$x=3$$

$\boxed{9-2x=-3}$

$$9-2x=-3$$
$$9-9-2x=-3-9$$
$$-2x=-12$$
$$\frac{-2x}{-2}=\frac{-12}{-2}$$
$$x=6$$

$$|5-2x|-11=0$$
$$|5-2x|=11$$

$$5-2x=-11$$
$$-2x=-11-5$$
$$-2x=-16$$
$$\frac{-2x}{-2}=\frac{-16}{-2}$$
$$x=8$$

$$5-2x=11$$
$$-2x=11-5$$
$$-2x=6$$
$$\frac{-2x}{-2}=\frac{6}{-2}$$
$$x=-3$$

1.6: Other Types of Equations:

$$-7|9-2x|+9=-12$$
$$-7|9-2x|=-21$$
$$|9-2x|=3$$

$9-2x=-3$	$9-2x=3$
$-2x=-3-9$	$-2x=3-9$
$-2x=-12$	$-2x=-6$
$\frac{-2x}{-2}=\frac{-12}{-2}$	$\frac{-2x}{-2}=\frac{-6}{-2}$
$x=6$	$x=3$

$$3|4-3x|+7=25$$
$$3|4-3x|=18$$
$$|4-3x|=6$$

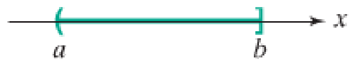
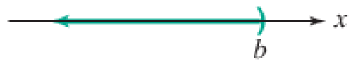
$4-3x=6$	$4-3x=-6$
$-3x=6-4$	$-3x=-6-4$
$-3x=2$	$-3x=-10$
$\frac{-3x}{-3}=\frac{2}{-3}$	$\frac{-3x}{-3}=\frac{-10}{-3}$
$x=-\frac{2}{3}$	$x=\frac{10}{3}$

Chapter1: Equations and Inequalities.

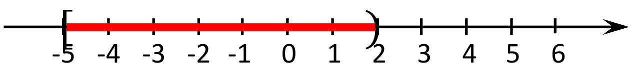
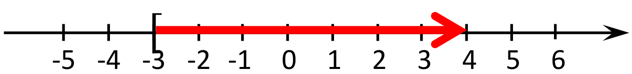
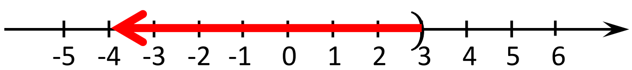
1.7: *Linear Inequalities and Absolute Value Inequalities:*

- ***Interval Notation:***

Let a and b be real numbers such that $a < b$.

Interval Notation	Set-Builder Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\{x \mid x$ is a real number$\}$ or $\mathbb{R}$ (set of all real numbers)	

Example1: Express the interval in set-builder notation and graph:

Interval	**Set-Builder notation**	**Graph**
$[-5, 2)$	$\{x \mid -5 \le x < 2\}$	
$[-3, \infty)$	$\{x \mid -3 \le x < \infty\}$	
$(-\infty, 3)$	$\{x \mid -\infty < x < 3\}$	

Chapter1: Equations and Inequalities.

1.7: *Linear Inequalities and Absolute Value Inequalities:*

Properties of Inequalities:

Property	Example
The Addition Property of Inequality If $a < b$, then $a + c < b + c$. If $a < b$, then $a - c < b - c$.	$2x + 3 < 7$ Subtract 3: $2x + 3 - 3 < 7 - 3$. Simplify: $2x < 4$.
The Positive Multiplication Property of Inequality If $a < b$ and c is positive, then $ac < bc$. If $a < b$ and c is positive, then $\frac{a}{c} < \frac{b}{c}$.	$2x < 4$ Divide by 2: $\frac{2x}{2} < \frac{4}{2}$. Simplify: $x < 2$.
The Negative Multiplication Property of Inequality If $a < b$ and c is negative, then $ac > bc$. If $a < b$ and c is negative, $\frac{a}{c} > \frac{b}{c}$.	$-4x < 20$ Divide by -4 and change the sense of the inequality: $\frac{-4x}{-4} > \frac{20}{-4}$. Simplify: $x > -5$.

- ***Solving Linear Inequalities in One Variable:***

Example2: Solve and graph the solution set on number line:

1) $3(x + 4) < 9x - 6$

2) $1 - (x + 3) \geq 4 - 2x$

3) $\frac{x-4}{6} \geq \frac{x-2}{9} + \frac{5}{18}$

4) $5(x-2) - 3(x+4) \geq 2x - 20$

$$5x - 10 - 3x - 12 \geq 2x - 20$$
$$2x - 22 \geq 2x - 20$$
$$2x - 2x \geq 22 - 20$$
$$0 \geq 2$$

No solution

- ***Solving Compound Inequalities:***

Example3: Solve and graph the solution set on number line:

1) $-11 < 2x - 1 \leq -5$

2) $-2 < 5 - 3x < 9$

1.7: *Linear Inequalities and Absolute Value Inequalities:*

• *Solving an Absolute Value Inequality:*

If u is an algebraic expression and c is a positive number,

1. The solutions of

$$|u| < c$$

are the numbers that satisfy $-c < u < c$.

ie: $|x| < 4 \Rightarrow -4 < x < 4$. Solution set is $(-4, 4)$

2. The solutions of

$$|u| > c$$

are the numbers that satisfy $u < -c$ or $c < u$.

ie: $|x| \geq 4 \Rightarrow x \leq -4$ *or* $4 \leq x$. Solution set is $(-\infty - 4] \cup [4, \infty)$

These rules are valid if $<$ is replaced by $\leq$ and $>$ is replaced by $\geq$.

Example4: Solve and graph the solution set on number line:

A) $|2x + 5| < 13$

Solution:

$$-13 < 2x + 5 < 13$$
$$-13 - 5 < 2x < 13 - 5$$
$$-18 < 2x < 8$$
$$-\frac{18}{2} < x < \frac{8}{2}$$
$$-9 < x < 4$$

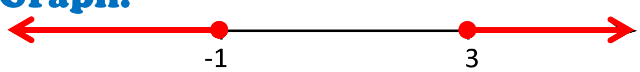
Interval: $(-9, 4)$ **Graph:**

B) $3|x - 1| + 2 \geq 8$

Solution:

$$3|x - 1| \geq 8 - 2$$
$$3|x - 1| \geq 6$$
$$\frac{3|x-1|}{3} \geq \frac{6}{3}$$
$$|x - 1| \geq 2$$
$$x - 1 \leq -2 \quad \text{or} \quad x - 1 \geq 2$$
$$x \leq -2 + 1 \quad \text{or} \quad x - 1 \geq 2 + 1$$
$$x \leq -1 \quad \text{or} \quad x \geq 3$$

Interval: $(-\infty, -1] \cup [3, \infty)$ **Graph:**



$C)\quad |2(x-1)+4| \leq 8$

Solution:

$$|2x-2+4| \leq 8$$

$$|2x+2| \leq 8$$

$$-8 \leq 2x+2 \leq 8$$

$$-8-2 \leq 2x \leq 8-2$$

$$-10 \leq 2x \leq 6$$

$$-\frac{10}{2} \leq \frac{2x}{2} \leq \frac{6}{2}$$

$$-5 \leq x \leq 3$$

Interval: $[-5, 3]$ **Graph:**

$D)\quad -2|3-2x| \geq -4$

$E)\quad 3 \leq |2x-1|$

Chapter2: Functions and Graphs

2.1: Basics of Functions and Their Graphs

Definition of a Relation

A relation is any set of ordered pairs. The set of all first components of the ordered pairs is called the ***domain*** of the relation and the set of all second components is called the ***range*** of the relation.

Example1: Find the domain and range of the relation:

$$\{(1,2),(3,4),(5,5),(-4,7),(-6,-2),(0,-3)\}$$

Definition of a Function

A function is a correspondence from a first set, called the domain, to a second set, called the range, such that each element in the domain corresponds to exactly one element in the range.

Example2: Determine whether each relation is a function. Give the domain and range for each relation.

A) $\{(1,4),(1,6),(1,-5)\}$

B) $\{(2,-2),(3,-2),(4,1),(2,9)\}$

C) $\{(3,-2),(5,3),(7,-1),(4,4)\}$

D) $\{(3,1),(5,1),(7,1),(4,1)\}$

Functions as Equations:

Functions are usually given in terms of equations rather than as sets of ordered pairs. The variable x is called the *independent* variable, and the variable y is called the *dependent* variable.

Note that: Not all Equation represent function!!

Solve each equation for y in term of x. If two or more values of y can be obtained for a given x, the equation is not a function.

Example3: Determine whether each equation defines y as a function of x:

a) $x^2 + y = 16$

b) $y^2 = x$

c) $x + y = 16$

d) $x + y^3 = 8$

e) $x = |y|$

f) $y = |x|$

Chapter2: Functions and Graphs
2.1: Basics of Functions and Their Graphs

Function Notation:

When the equation represents a function, the function is often named by a letter such as f, g, F, G.

Suppose that f names a function. The special notation $f(x)$, read "f of x" or "f at x" , represents the value of function at the number x.

For example:

If $y = x^2 + 3x + 5$, then $y = f(x) = x^2 + 3x + 5$.

Example4: If $h(x) = x^2 - 3x$, then evaluate

a) $h(2) =$

b) $h(-1) =$

c) $h(-3) =$

d) $h(4)$

Example5: If $f(x) = 3 - 2x$, evaluate

a) $f(-6) =$

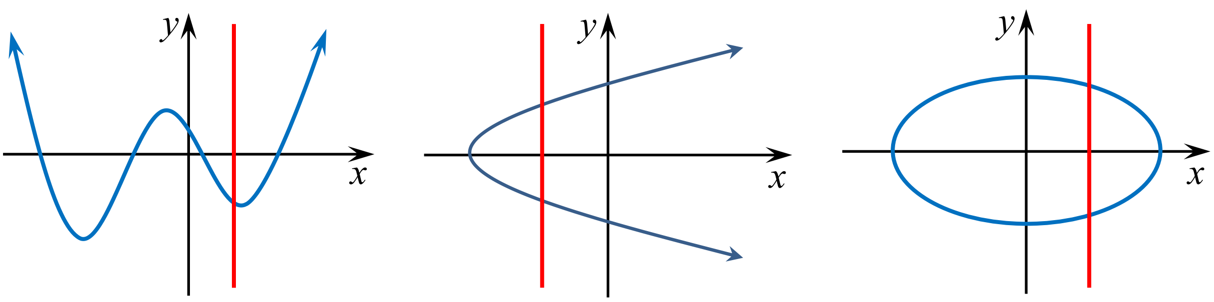
c) $f(3) =$

b) $f(4) =$

d) $f(-2)$

The Vertical Line Test:

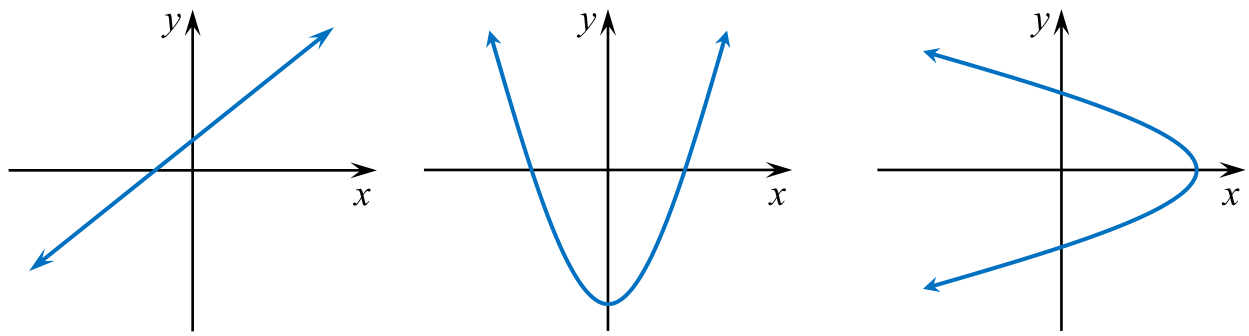
If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.



The first graph is a function, the second and the third one are not.

Chapter2: Functions and Graphs
2.1: Basics of Functions and Their Graphs

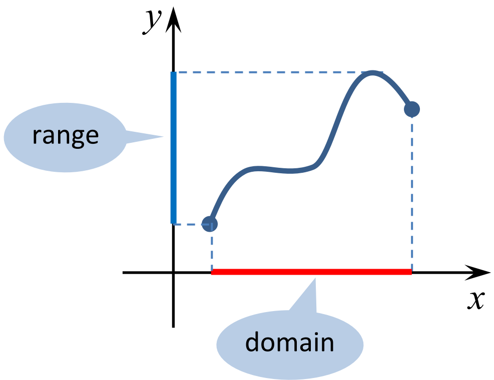
Example 6: Use the vertical line test to identify graphs in which y is a function of x.

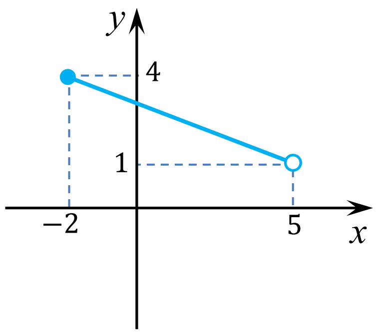


Identifying Domain and Range from a Function's Graph.

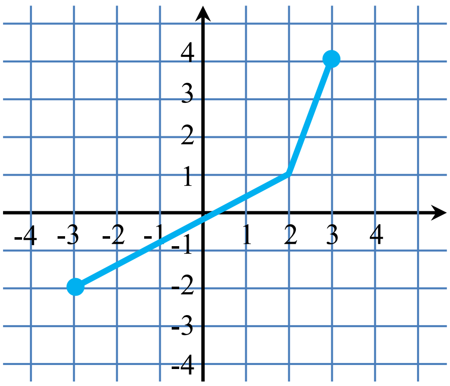
Domain: set of inputs (Found on the x-axis)

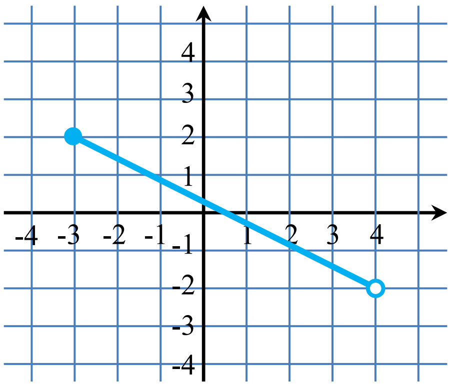
Range: set of outputs (Found on the y-axis)

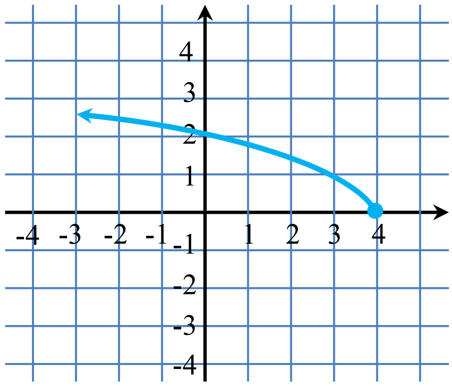




Example7: Use the graph of each function to identify its domain and its range.







Domain	
Range	

Domain	
Range	

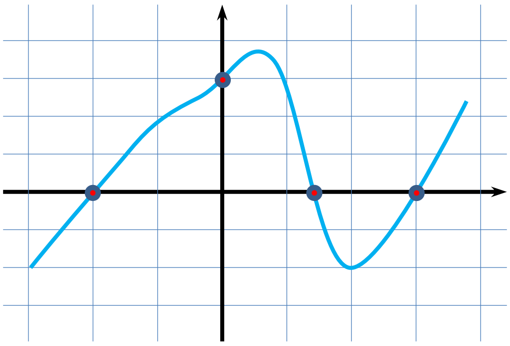
Domain	
Range	

Chapter2: Functions and Graphs
2.1: Basics of Functions and Their Graphs

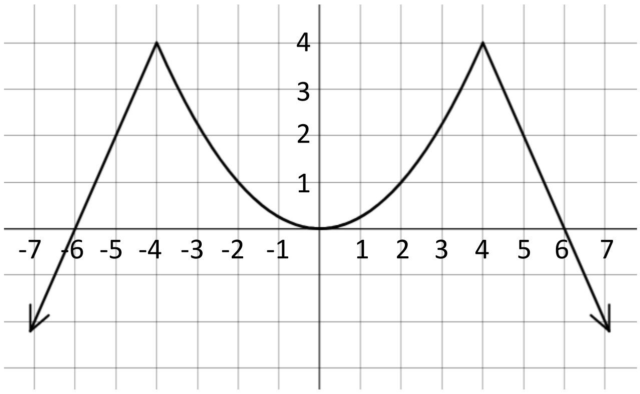
Identifying Intercepts from a Function's Graph:

***x*-intercept (the zeros of the function):** the points at which the graph crosses the *x*-axis.

***y*-intercept:** the points at which the graph crosses the *y*-axis.



Example8: Use the graph of f to find each indicated function value.



1) Find the x-intercepts

2) Find the y-intercepts

3) $f(-5) =$

5) $f(4) =$

6) $f(2) =$

7) Domain

8) Range

Example9: Use the graph to find:

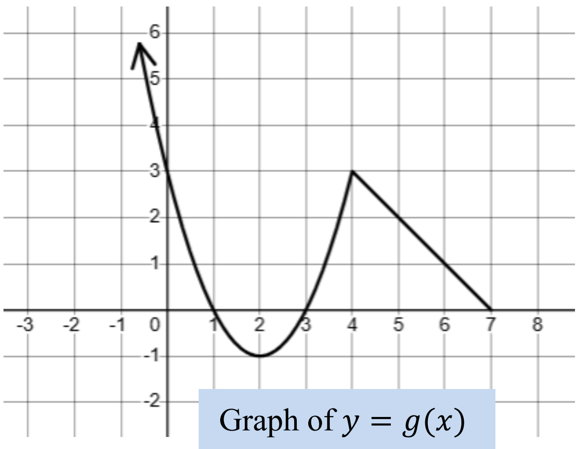
1) Domain

2) Range

3) x-intercept

4) y-intercept

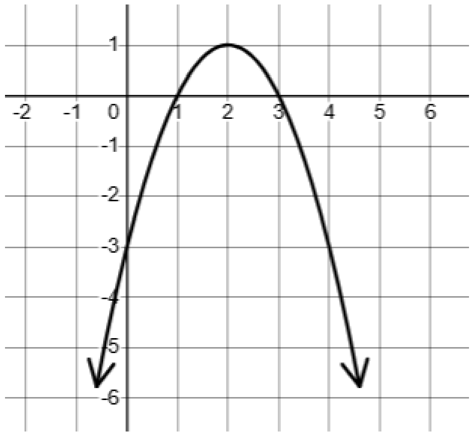
5) $g(4) =?$ $\qquad g(x) = -1 \Rightarrow x = ?$

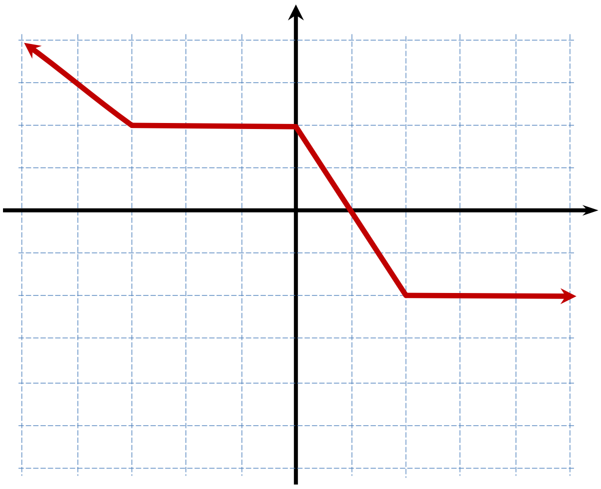


Chapter2: Functions and Graphs
2.1: Basics of Functions and Their Graphs

Example 10: Use the graph to complete the table

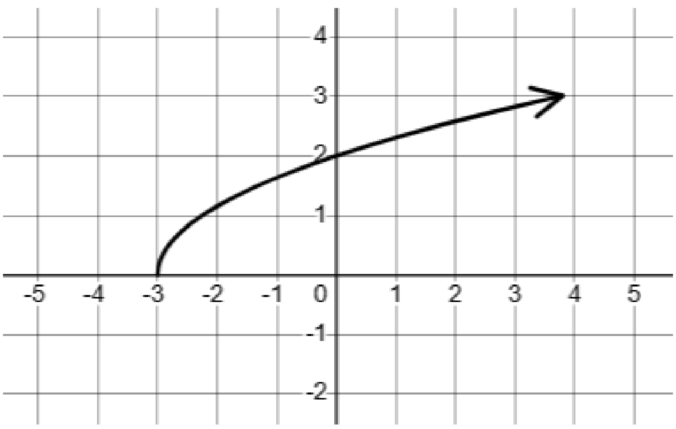
Domain	
Range	
x-intercept	
y-intercept	
$f(-2) =$	
$f(4) =$	





Domain	
Range	
x-intercept	
y-intercept	
$f(-4) =$	
$f(4) =$	

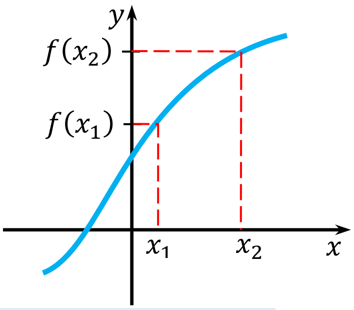
Domain	
Range	
x-intercept	
y-intercept	
$f(0) =$	



2.2 More on Functions and Their Graphs

Increasing, Decreasing, and Constant Functions

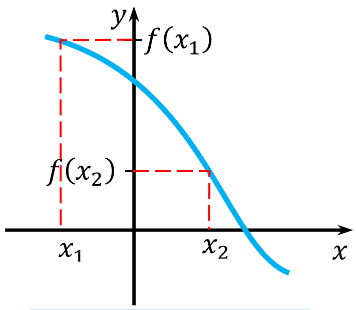
I. A function is *increasing* on an open interval I, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
2. A function is *decreasing* on an open interval, I, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
3. A function is constant on an open interval. if $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval.



For $x_1 < x_2$ in I,

$f(x_1) < f(x_2)$;

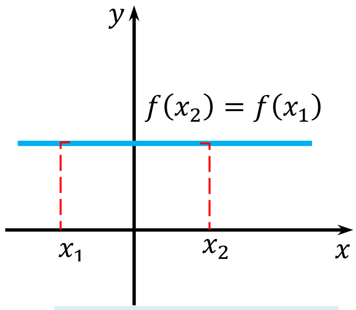
f is *increasing* on I



For $x_1 < x_2$ in I,

$f(x_1) > f(x_2)$;

f is *decreasing* on I



For any x_1, x_2 in I,

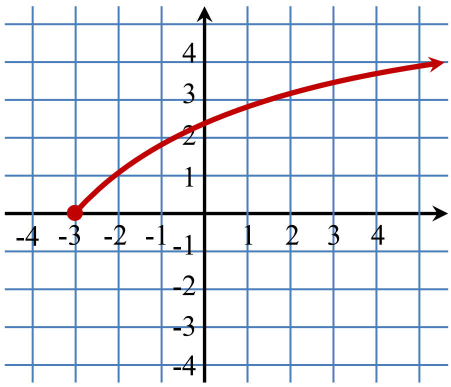
$f(x_1) = f(x_2)$;

f is *constant* on I

Use the graph to determine:
a. intervals on which the function is increasing, if any.

b. intervals on which the function is decreasing, if any.

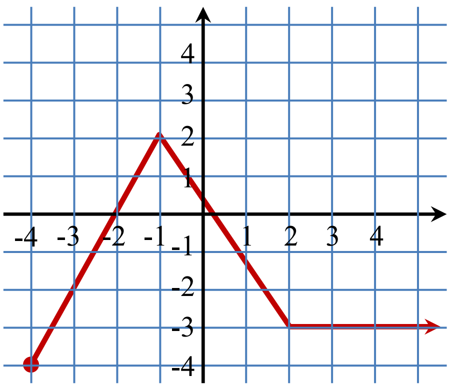
c. intervals on which the function is constant, if any.



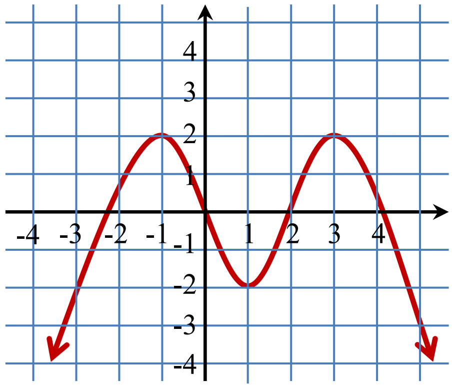
a. intervals on which the function is increasing, if any.

b. intervals on which the function is decreasing, if any.

c. intervals on which the function is constant, if any.



2.2 More on Functions and Their Graphs



a. intervals on which the function is increasing, if any.

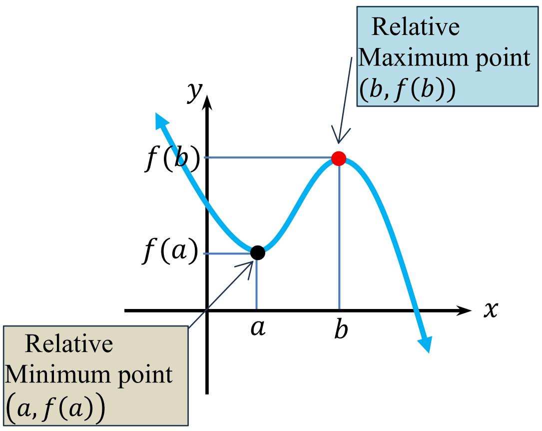
b. intervals on which the function is decreasing, if any.

c. intervals on which the function is constant, if any.

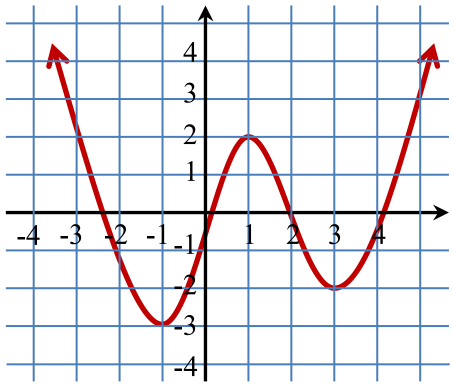
Definitions of Relative Maximum and Relative Minimum

1. A function value $f(a)$ is a **relative maximum** of f if there exists an open interval containing a such that $f(a) > f(x)$ for all $x \neq a$ in the open interval.

2. A function value $f(b)$ is a **relative minimum** Of f if there exists an open interval containing b such that $f(b) < f(x)$ for all $x \neq b$ in the open interval.



Example1. The graph of a function f is given. Use the graph to find each of the following:



a) The numbers, at which f has a relative maximum.

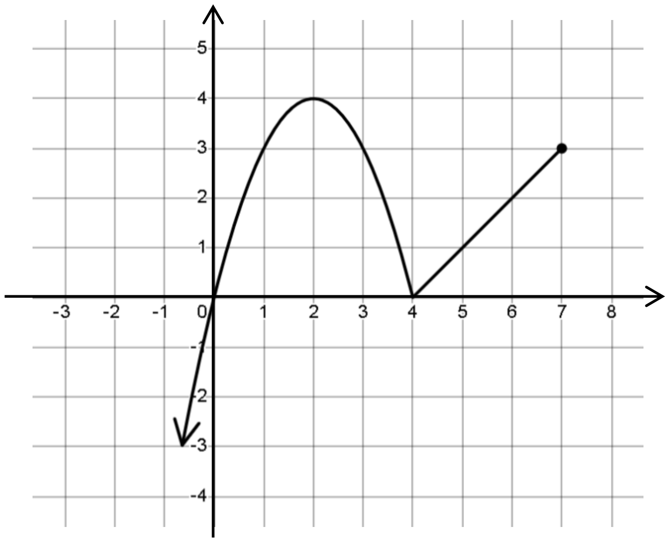
b) What are these relative maxima?

c) The numbers,at which f has a relative minimum.

d) What are these relative minima?

2.2 More on Functions and Their Graphs

Example2. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing.

f. intervals on which f is decreasing.

g. intervals on which f is constant.

h. the number at which f has a relative minimum.

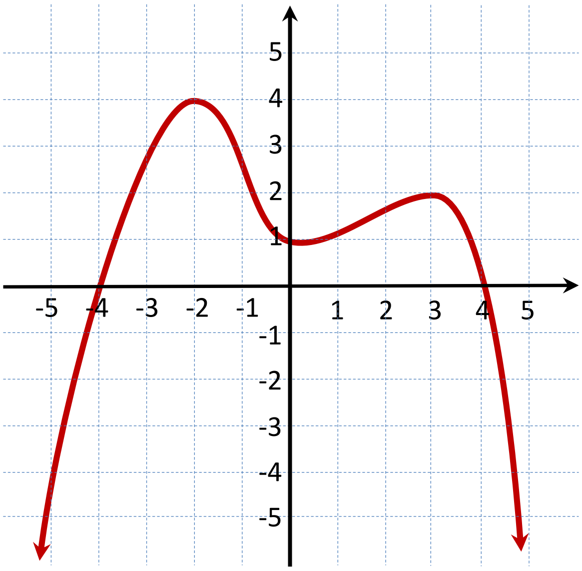
i. the relative minimum of f

j. $f(6) =$

k. the values of x for which $f(x) = 3$

2.2 More on Functions and Their Graphs

Example3. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing.

f. intervals on which f is decreasing.

g. the number at which f has a relative maximum.

h. the relative maxima of f

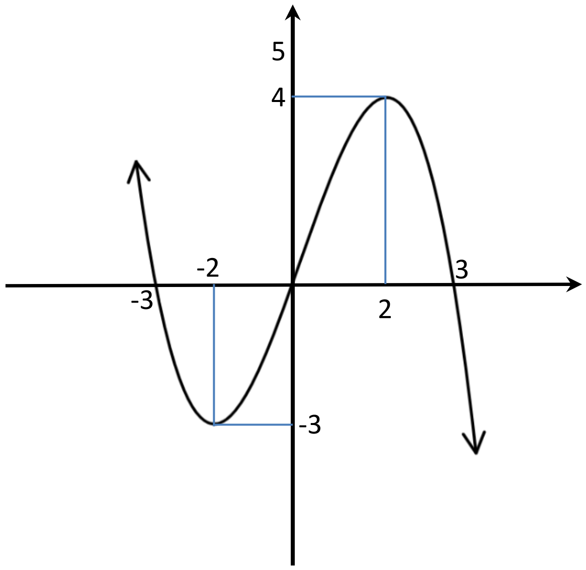
i. $f(-2) =$

j. the values of x for which $f(x) = 0$

2.2 More on Functions and Their Graphs

Example4. Use the graph of f to determine each of the following. Where applicable, use interval notation.

a. the domain of f



b. the range of f

c. the x-intercepts (zeros of f)

d. the y-intercept ($f(0) = ?$)

e. intervals on which f is increasing.

f. intervals on which f is decreasing.

g. the number at which f has a relative maximum.

h. the relative maxima of f

i. is $f(-1)$ positive or negative

j. the values of x for which $f(x) = 4$

Example5. Use the graph of f to determine each of the following. Where applicable, use interval notation.

a. the domain of f

b. the range of f

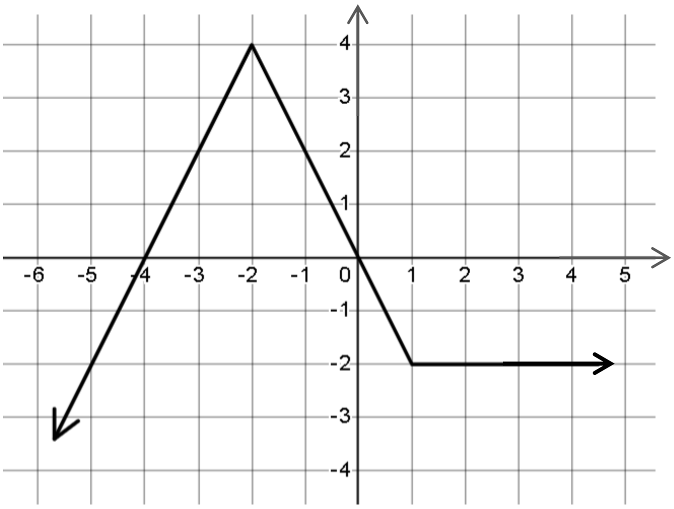
c. the x-intercepts (zeros of f)

d. the y-intercept ($f(0) = ?$)

e. intervals on which f is increasing.

f. intervals on which f is decreasing.

g. intervals on which $f(x) \leq 0$.



h. the number at which f has a relative maximum.

i. the relative maxima of f

j. is $f(-1)$ positive or negative

k. the values of x for which $f(x) = 4$

Example6. Evaluate each piecewise function at the given values of the independent variable.

1) $f(x) = \begin{cases} 3x+5 & \text{if } x < 0 \\ 4x+7 & \text{if } x \geq 0 \end{cases}$

$a)\ f(-2) =$

$b)\ f(0) =$

$c)\ f(3) =$

2) $g(x) = \begin{cases} x+3 & \text{if } x \geq -3 \\ -(x+3) & \text{if } x < -3 \end{cases}$

$a)\ g(0) =$

$b)\ g(-6) =$

$c)\ g(-3) =$

3) $h(x) = \begin{cases} x^2-9 & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

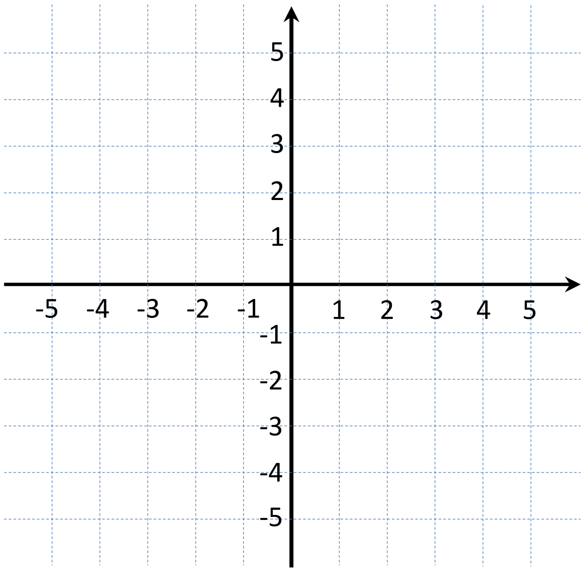
$a)\ h(5) =$

$b)\ h(0) =$

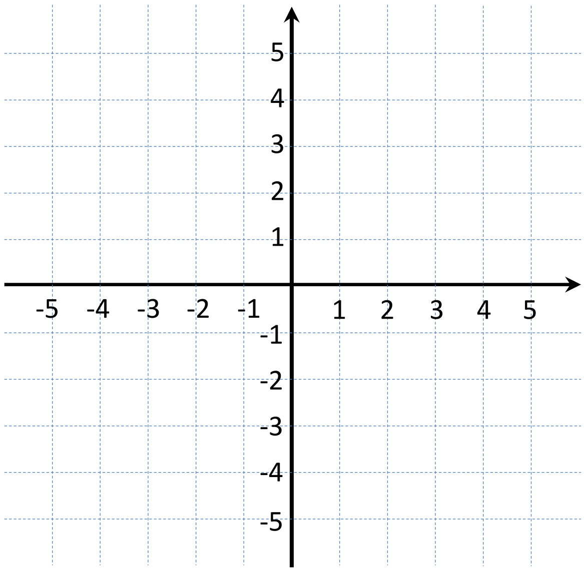
$c)\ h(3) =$

Example7. Graph the function and use the graph to find the range of the function.

1) $f(x) = \begin{cases} -x+2 & \text{if } x < -1 \\ x-1 & \text{if } x \geq -1 \end{cases}$



2) $f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ x-2 & \text{if } x \geq 1 \end{cases}$



Chapter 2: Functions and Graphs

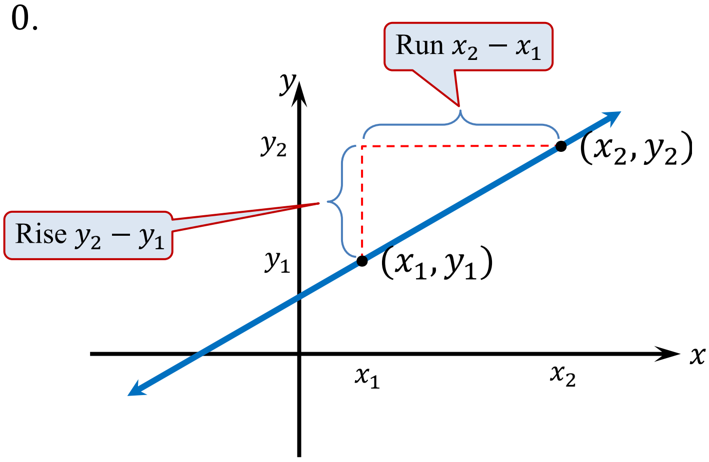
2.3: Linear Functions and Slope

Definition of Slope

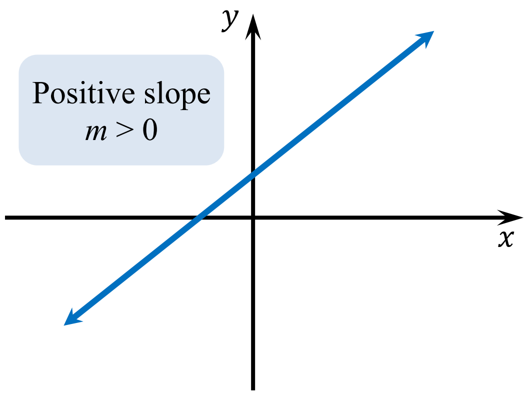
The slope of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

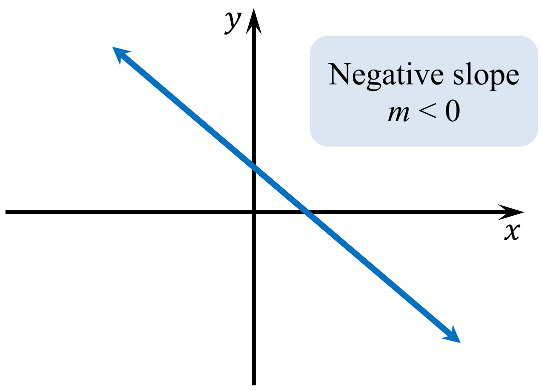
$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

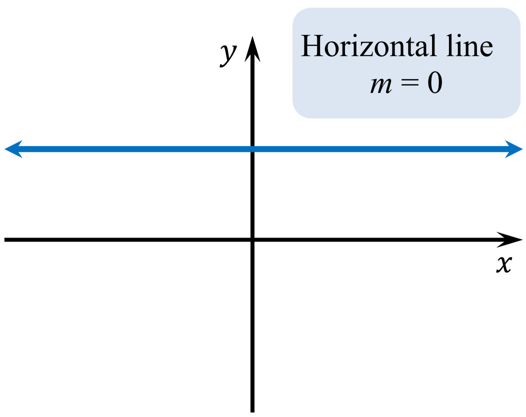
where $x_2 - x_1 \neq 0$.

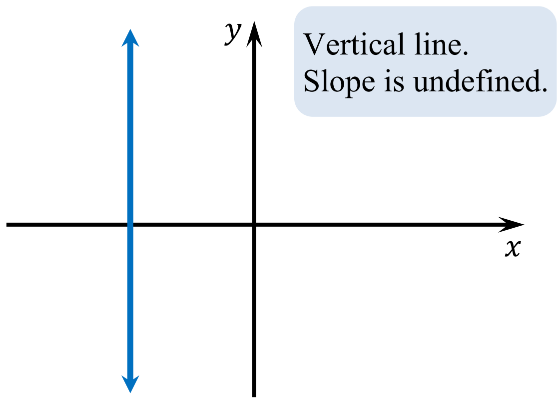


Possibilities for a Line's Slope









Chapter 2: Functions and Graphs

2.3: Linear Functions and Slope

Example 1.

Find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

a) $(3, -4)$ and $(3, 5)$

b) $(-3, 4)$ and $(2, -2)$

c) $(4, -1)$ and $(-4, -1)$

d) $(-1, 3)$ and $(2, 4)$

Chapter 2: Functions and Graphs

2.3: Linear Functions and Slope

Point-Slope Form of the Equation of a Line

The *point-slope form of the equation* of a nonvertical line with slope m that passes through the point (x_1, y_1) is given by

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

Example 2.

Write an equation in ***point-slope*** form for the line with slope 4 that passes through the point $(-1, 3)$. Then solve the equation for *y*.

Slope-Intercept Form of the Equation of a Line

The *slope-intercept form* of the equation of a nonvertical line with slope m and *y-intercept b* is given by

$$\boldsymbol{y = mx + b}$$

Example 3. Use the given conditions to write an equation for each line in ***Point-Slope***, and ***Slope-intercept forms***.

A) Write an equation in ***point-slope*** and ***slope-intercept*** form with slope $= -2$, and passing through $(0, -3)$

B) Write an equation in ***point-slope*** and ***slope-intercept*** form passing trough $(-2, -5)$ and $(6, -5)$

C) Write an equation in ***point-slope*** and ***slope-intercept*** form with *x*-intercept $= 4$ and *y*-intercept $= -2$

Chapter 2: Functions and Graphs

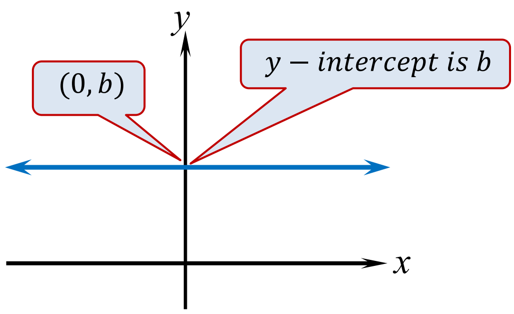
2.3: Linear Functions and Slope

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where b is the y-intercept of the line. The slope of a horizontal line is zero.



Because any vertical line can intersect the graph of a horizontal line $y = b$ only once, a horizontal line is the graph of a function. Thus. we can express the equation $y = b$ as $f(x) = b$. This linear function is often called a ***constant function.***

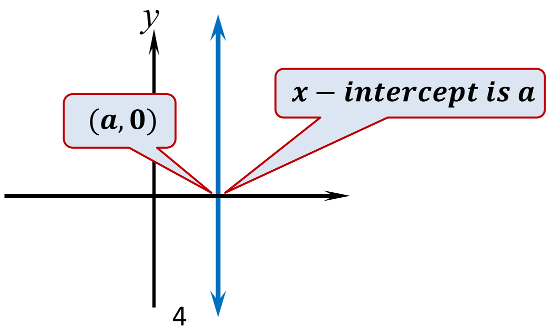
Example 4. Graph $f(x) = 1$

Equations of Vertical and Horizontal Lines

A vertical line is given by an equation of the form

$$x = a$$

where a is the x-intercept of the line. The slope of a vertical line is undefined.

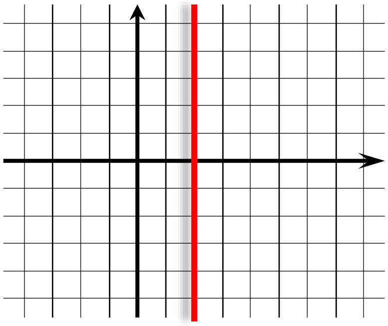


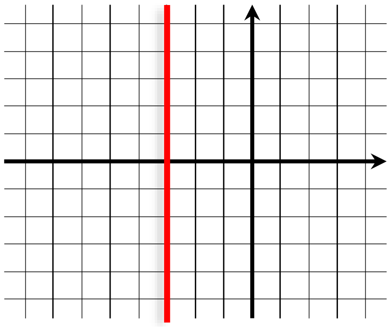
Chapter 2: Functions and Graphs

2.3: Linear Functions and Slope

No Vertical line represents a Linear Function.

Example 5. Graph $x = 2$ Graph $x = -3$



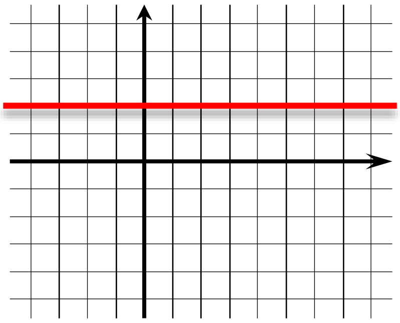


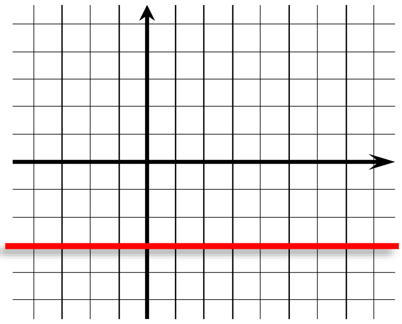
A horizontal line is given by an equation of the form

$$y = a$$

The slope of a horizontal line is zero.

Example 6. Graph $y = 2$ Graph $y = -3$



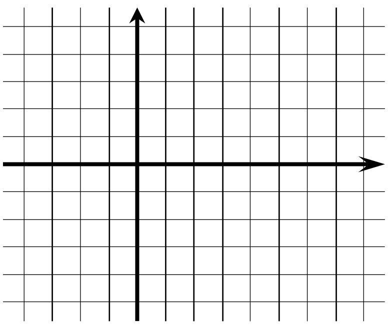


Using Intercepts to Graph $Ax + By + C = 0$

1) Find the $x - intercept$. Let $y = 0$ and solve for x. Plot the point containing $x - intercept$ on the $x - axis$.
2) Find the $y - intercept$. Let $x = 0$ and solve for y. Plot the point containing the $y - intercepy$ on the $y - axis$.

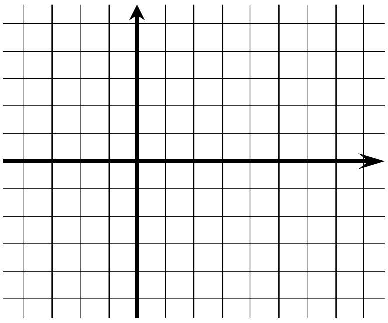
Example 7. Use intercepts to graph the equations.

$A)$ $6x - 9y - 18 = 0$

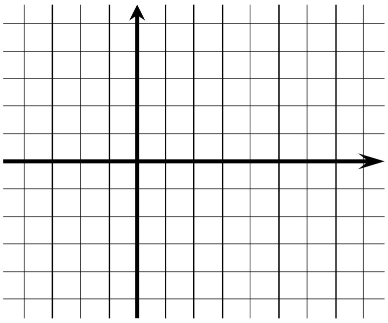


2.3: Linear Functions and Slope

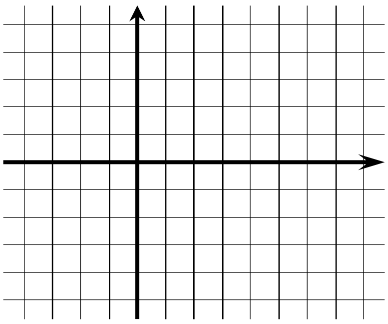
$B)\ 6x - 3y + 12 = 0$



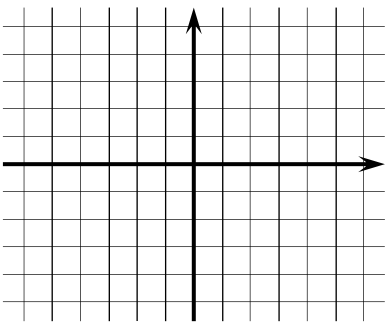
$C)\ 4y + 28 = 0$



$D)\ 3x - 18 = 0$



$E)\ 3x - 4y - 12 = 0$



2.4: More on Slope

Parallel Lines:

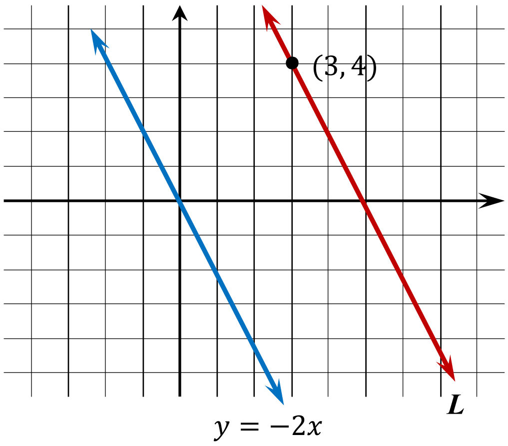
Two non-intersecting lines that lie in the same plane.
If two nonvertical lines are parallel, then they have the **same slope**.

$$\boldsymbol{m_1 = m_2}$$

Example 1.

Write an equation for line L in *point-slope form* and *slope-intercept form*.

L is parallel to $y = -2x$



Example 2.

Use the given conditions to write an equation for each line in *point-slope* form and *slope-intercept* form.

a) Write an equation of the line (in **point-slope** and **slope-intercept**) forms passing through $(-2, -7)$ and **parallel** to the line whose equation is $y = -5x + 4$

b) Write an equation of the line (in **point-slope** and **slope-intercept**) forms passing through $(2, -3)$ and **parallel** to the line whose equation is $2x - 5y + 10 = 0$

c) Write an equation of the line (in **point-slope** and **slope-intercept**) forms passing through $(-2, 5)$ and **parallel** to the line who passes through the points $(2, -2)$ and $(3, -5)$

2.4: More on Slope

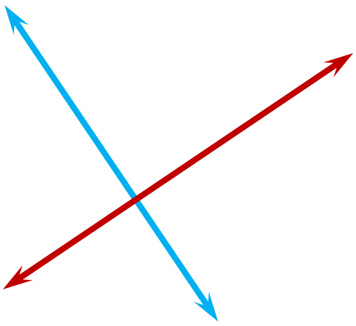
Perpendicular Lines:

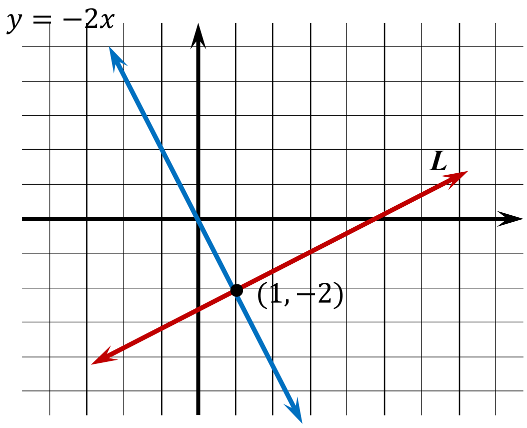
Two lines that intersect at a right angle 90° are said to be perpendicular.
An equivalent way of stating this relationship is to say that one line is perpendicular to another line if its slope is the negative reciprocal of the slope of the other line.
For example, if a line has slope 5, any line having slope $-\frac{1}{5}$ is perpendicular to it.
Similarly. if a line has slope $-\frac{3}{4}$, any line having slope $\frac{4}{3}$ is perpendicular to it.

$$\boxed{m_1 = -\frac{1}{m_2}}$$

Example 3.

Find an equation for the line ***L***





L is perpendicular to $y = -2x$

Example 4.

Write an equation for the line in ***point-slope*** form and ***slope-intercept*** form.

a) Write an equation of the line (in **point-slope** and **slope-intercept**) forms passing through $(-4, 2)$ and **perpendicular** to the line whose equation is $y = \frac{1}{3}x + 7$

b) Write an equation of the line (in **point-slope** and **slope-intercept**) forms passing through $(5, -9)$ and **perpendicular** to the line whose equation is $x + 7y - 12 = 0$

Chapter 2: Functions and Graphs

2.4: More on Slope

Summary

How to find slope of lines		
Equation	**Slope**	**Graph**
$y = c$	$m = 0$	Horizontal line passing through $(0, c)$
$x = c$	Slope is undefined	Vertical line passing through $(c, 0)$
$y = mx + b$	Slope is m	Slanted line
$ax + by + c = 0$	Slope is $-\frac{a}{b}$	Slanted line

Chapter 2: Functions and Graphs

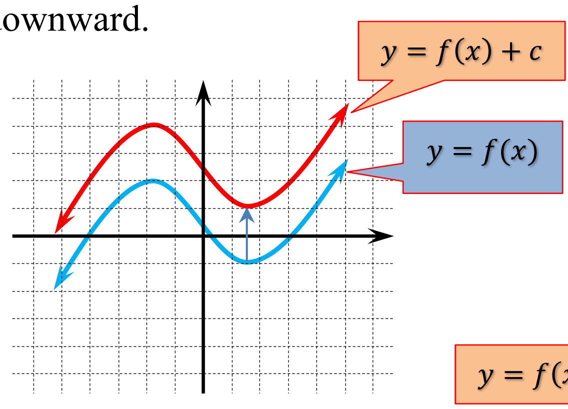
2.5 Transformations of Functions

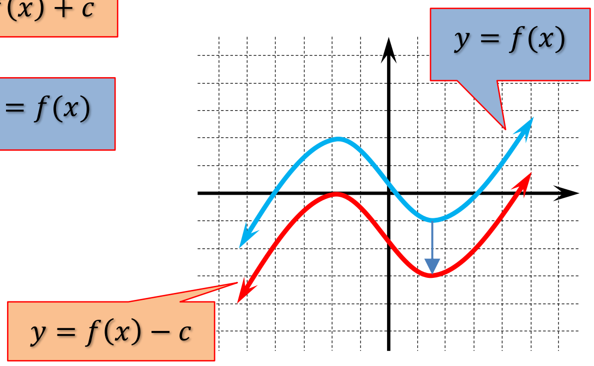
Transformations of Functions:

1)Vertical Shifts:

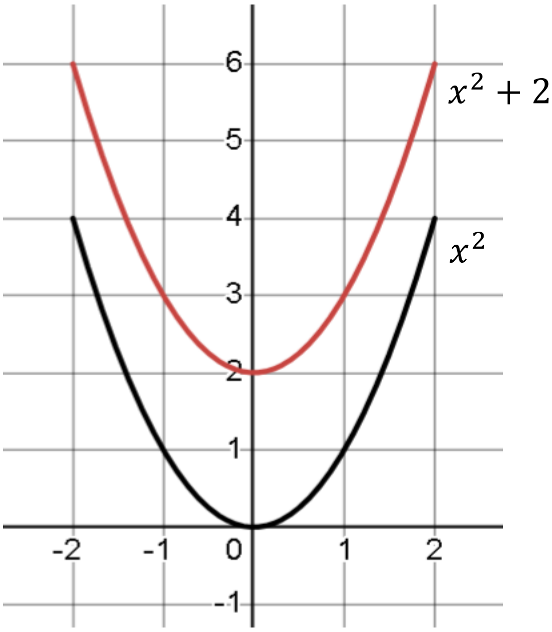
Let f be a function and c a positive real number.

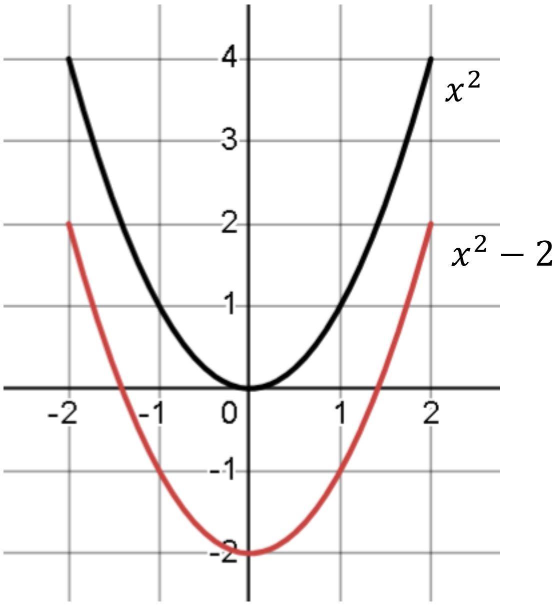
- The graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted c units vertically upward.
- The graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted c units vertically downward.





Example1: Use the graph $f(x) = x^2$ to obtain the graph of $g(x) = x^2 + 2$.





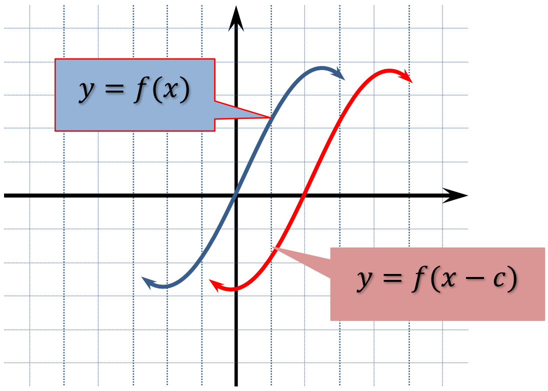
Chapter 2: Functions and Graphs

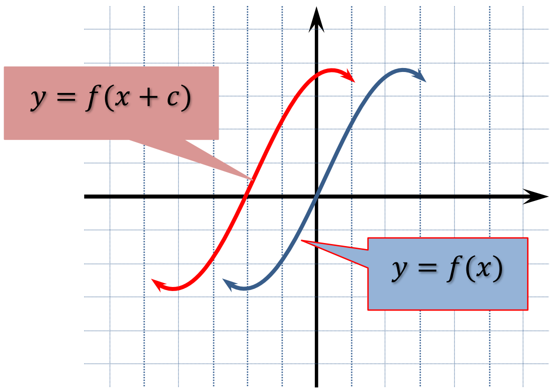
2.5 Transformations of Functions

2) Horizontal Shifts:

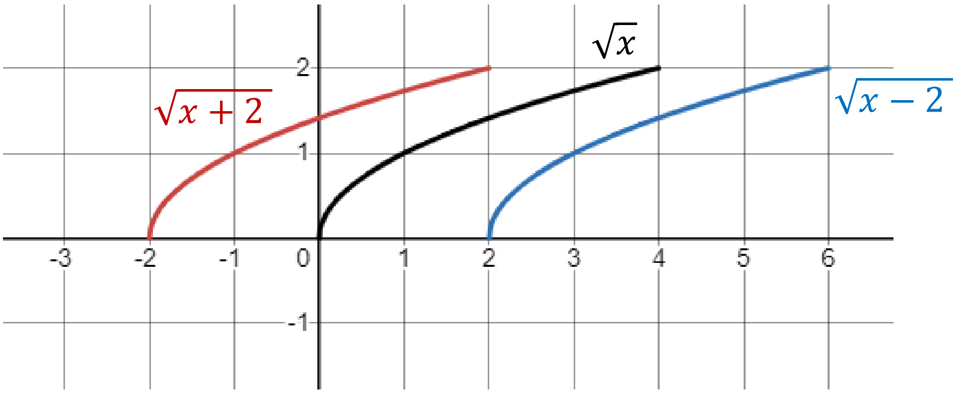
Let f be a function and c a positive real number.

❄ The graph of $y = f(x+c)$ is the graph of $y = f(x)$ shifted to the left c units.
❄ The graph of $y = f(x-c)$ is the graph of $y = f(x)$ shifted to the right c units.

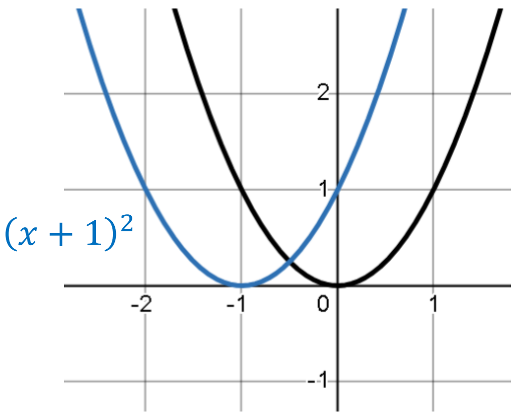


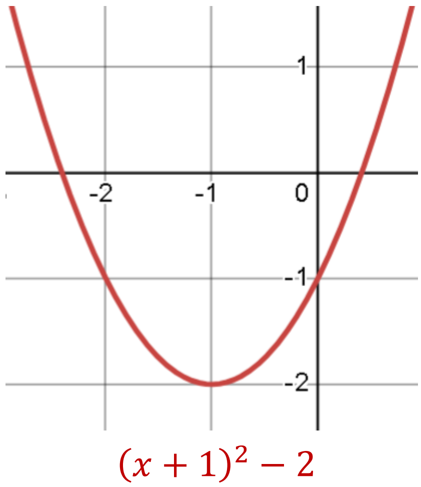


Example2: Use the graph $f(x) = \sqrt{x}$ to obtain the graph of $g(x) = \sqrt{x+2}$.



Example3: Use the graph $f(x) = x^2$ to obtain the graph of $g(x) = (x+1)^2 - 2$.





Chapter 2: Functions and Graphs

2.5 Transformations of Functions

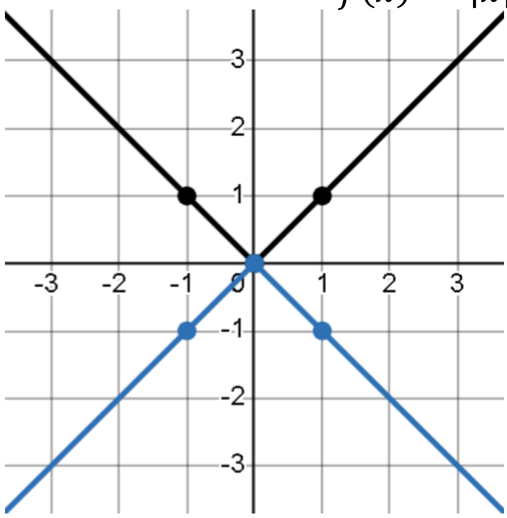
3) Reflections of Graphs about axis:

Reflection about the $x-$axis.

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis.

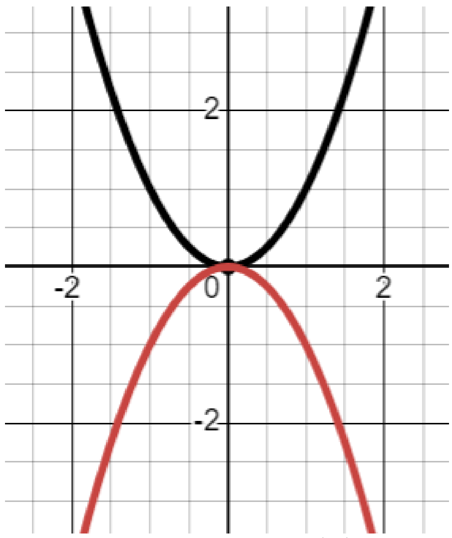
Example4: Use the graph $f(x)$ to obtain the graph of $g(x)$.

$f(x) = |x|$

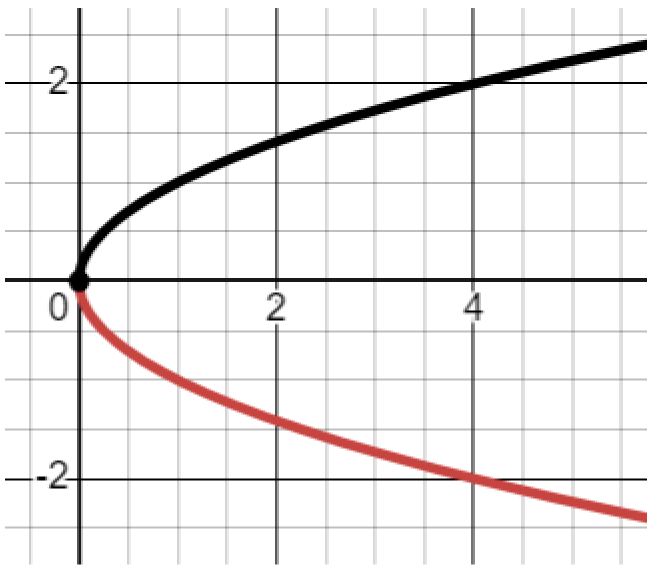


$g(x) = -|x|$

$f(x) = x^2$



$g(x) = -x^2$



$f(x) = \sqrt{x}$

$g(x) = -\sqrt{x}$

Chapter 2: Functions and Graphs

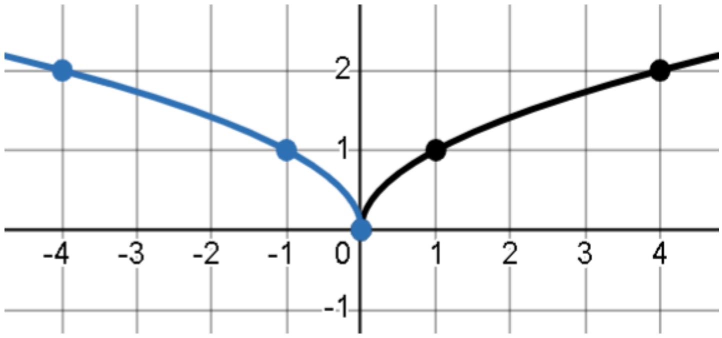
2.5 Transformations of Functions

Reflection about the $y-$ axis.

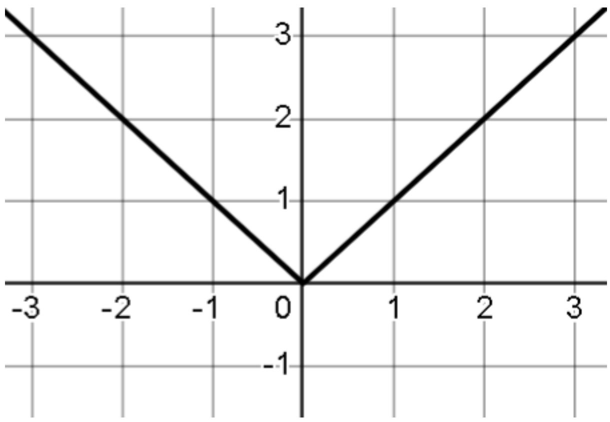
The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about the y-axis.

Example 5: Use the graph $f(x)$ to obtain the graph of $g(x)$.

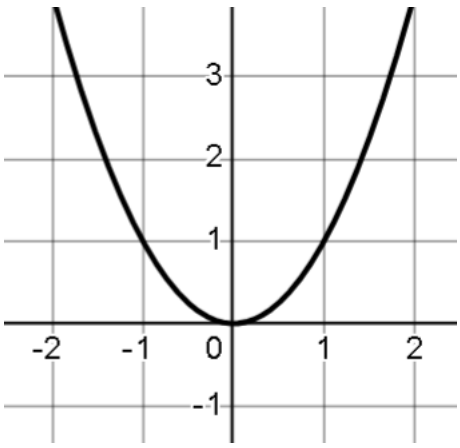
$g(x) = \sqrt{-x}$



$f(x) = \sqrt{x}$



$f(x) = |x| = |-x| = g(x)$



$f(x) = x^2 = (-x)^2 = g(x)$

Chapter 2: Functions and Graphs

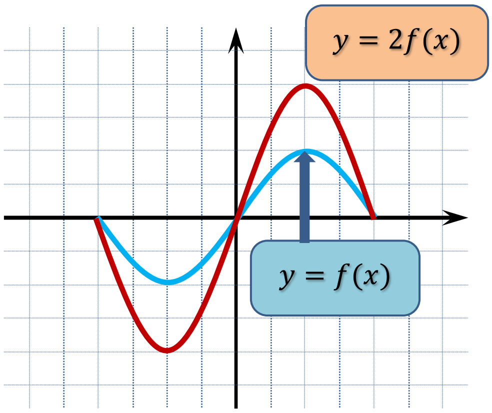
2.5 Transformations of Functions

4) Vertical Stretching and Shrinking:

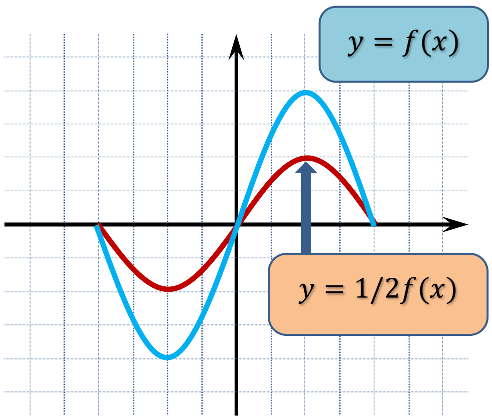
Vertically Stretching and Shrinking Graphs

Let f be a function and c a positive real number.

- If $c > 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ *vertically stretched* by multiplying each of its y-coordinates by c.
- If $0 < c < 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ *vertically shrunk* by multiplying each of its y-coordinates by c.

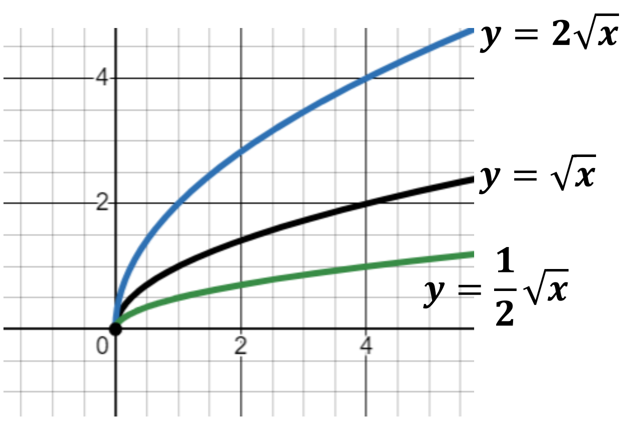


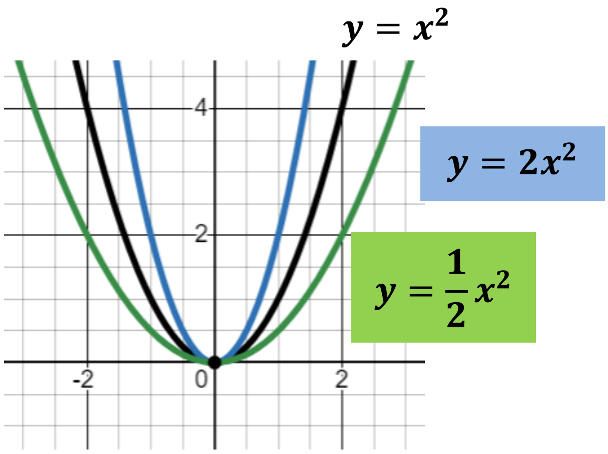
Vertical Stretching

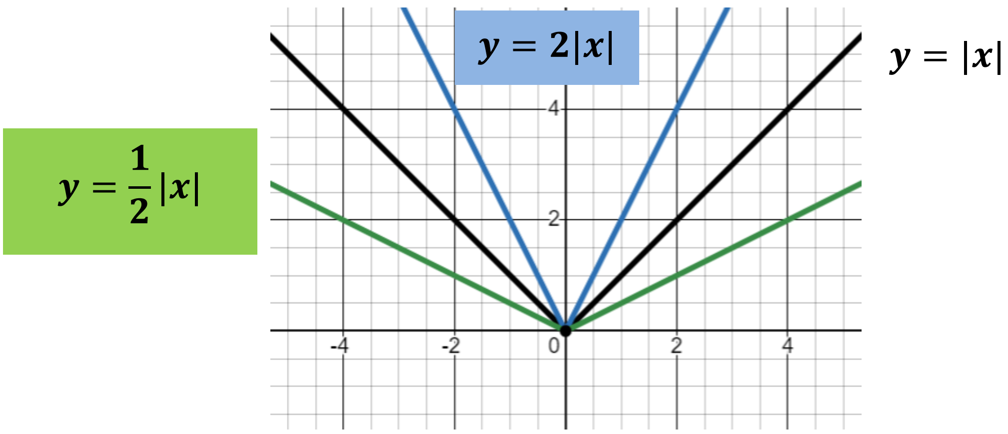


Vertical Shrinking

Example6:



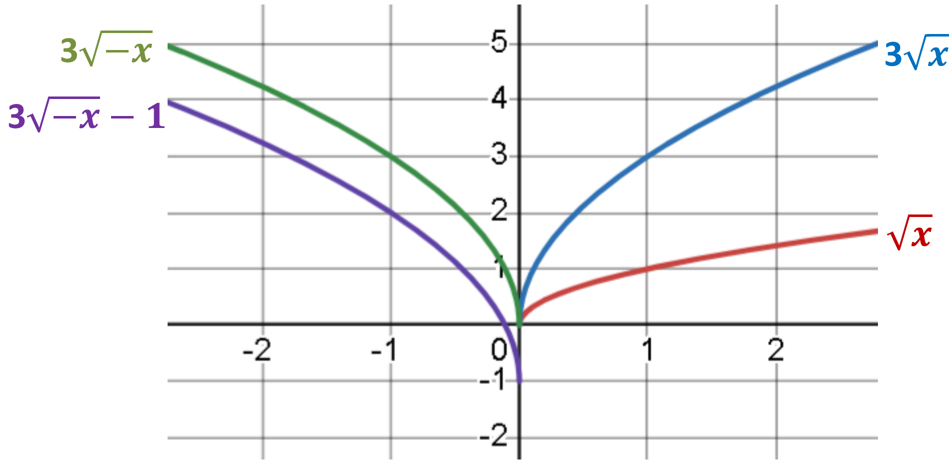




Chapter 2: Functions and Graphs

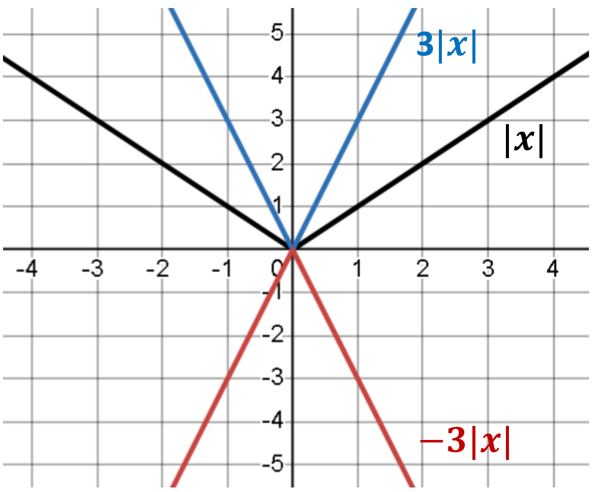
2.5 Transformations of Functions

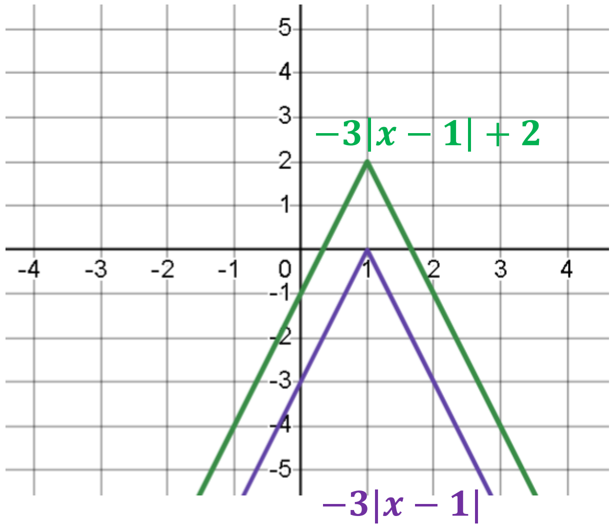
Example 7: Use the graph $f(x) = \sqrt{x}$ to obtain the graph of $h(x) = 3\sqrt{-x} - 1$.



Example 8: Use the graph $f(x) = |x|$ to obtain the graph of

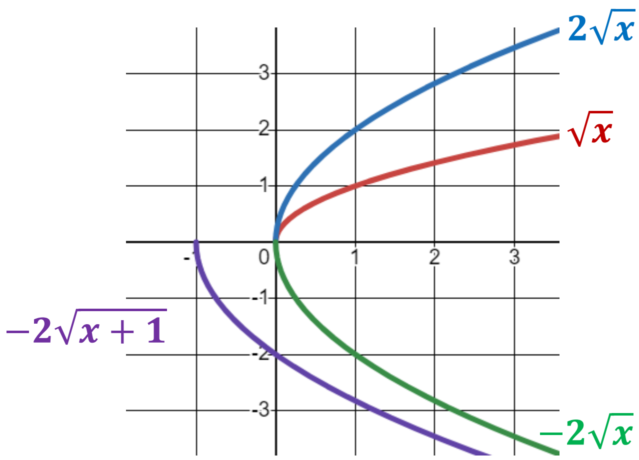
$g(x) = -3\,|x - 1\,| + 2$

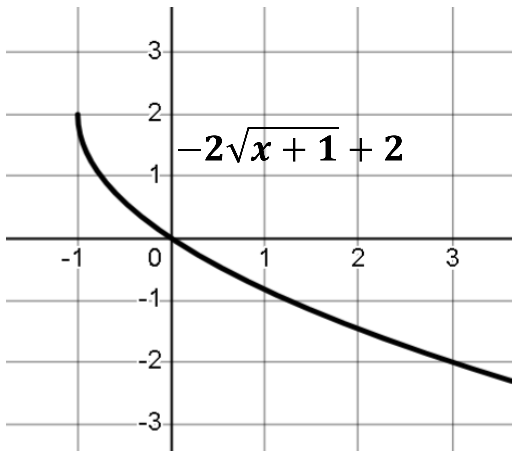




Example 9: Use the graph $f(x) = \sqrt{x}$ to obtain the graph of

$g(x) = -2\sqrt{x+1} + 2$

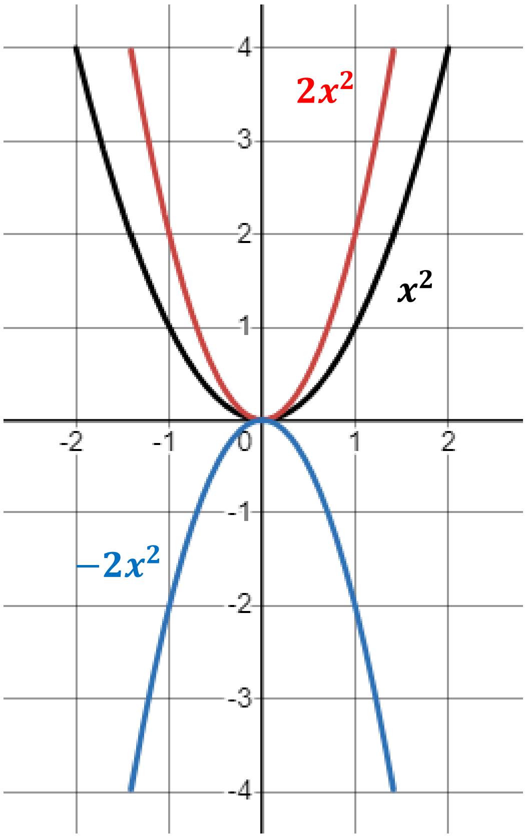


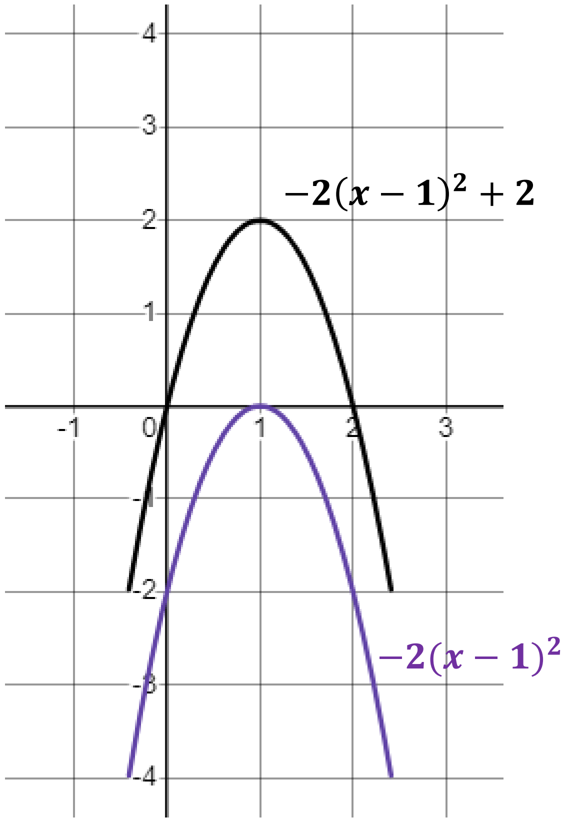


2.5 Transformations of Functions

Example 10: Use the graph $f(x) = x^2$ to obtain the graph of

$g(x) = -2(x-1)^2 + 2$





2.6 Combinations of Functions; Composite Functions.

● The Domain of a Function:

Domain is the largest set of real numbers for which the value of $f(x)$ is a real number.

- If $f(x)$ is a **polynomial** then domain f is the set of all real numbers R
- If $f(x)$ is a **rational function** then domain f is
$$R - \{zeros\ of\ the\ denominator\}$$
- If $f(x) = \sqrt{g(x)}$, the domain $of\ f$ is $g(x) \geq 0$
- If $f(x) = \frac{r(x)}{\sqrt{g(x)}}$, the domain of f is $g(x) > 0$, where $r(x) is\ a\ polynomial$

Example1: Find the domain of each function

1) $f(x) = x^2 - 7x$

2) $f(x) = \sqrt{24 - 2x}$

3) $h(x) = \sqrt{3x + 12}$

4) $g(x) = \frac{3x+2}{x^2-2x-3}$

2.6 Combinations of Functions; Composite Functions.

5) $g(x) = \frac{2}{x+5}$

6) $g(x) = \frac{1}{x^2+1} - \frac{1}{x^2-1}$

7) $g(x) = \frac{\sqrt{x-3}}{x-6}$

8) $g(x) = \frac{1}{\sqrt{x+2}}$

2.6 Combinations of Functions; Composite Functions.

The Algebra of Functions:
Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The sum $f+g$, the difference $f-g$, the product fg, and the quotient $\frac{f}{g}$ are functions whose domains are the set of all numbers common to the domains of f and g $(D_f \cap D_g)$, defined as follows:

1. Sum: $(f+g)(x) = f(x) + g(x)$

2. Difference: $(f-g)(x) = f(x) - g(x)$

3. Product: $(fg)(x) = f(x)g(x)$

4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

Example1:

Find $f+g$, $f-g, f \cdot g$ and f/g. Determine the domain for each function.

35. $f(x) = 2x^2 - x - 3$, $g(x) = x + 1$

$f(x) = \sqrt{x+1}$, $\quad g(x) = \sqrt{4-x}$

2.6 Combinations of Functions; Composite Functions.

Example 2: Let $f(x) = 3x^2 - 2 \ and \ g(x) = \sqrt{x}$,then calculate

A) $(f + g)(x)$

B) Domain of $f + g, f - g, f . g$

The Composition of Functions

The composition of the functions f and g is denoted by $f \circ g$ and is defined by
the equation

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)(x)$ = domain of the result ∩ domain of g

Example 3: If $f(x) = 3x \, , g(x) = x - 5$, then find

$i)$ $(f \circ g)(x)$

$ii)$ $(f \circ g)(2)$

2.6 Combinations of Functions; Composite Functions.

Example 4: If $f(x) = x^2 + 2, g(x) = x + 1$ find

$i)\ (g \circ f)(x)$

$ii)\ (g \circ f)(-1)$

Example 5: If $f(x) = \sqrt{x}, g(x) = x - 2$ find

$i)\ (f \circ g)(x)$

$ii)\ domain\ (f \circ g)$

$(g \circ f)(x)$

$ii)\ domain\ (g \circ f)$

2.6 Combinations of Functions; Composite Functions.

Example 6: If $f(x) = x^2 + 4,\ g(x) = \sqrt{x}$ find

$i)\ (f \circ g)(x)$

$ii)\ domain\ (f \circ g)$

2.7: Inverse Function.

Definition of the Inverse of a Function

Let f and g be two functions such that

$$f(g(x)) = x \text{ for every } x \text{ in the domain of } g,$$

and

$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f.$$

The function g is the ***inverse of the function*** f and is denoted by f^{-1} (read"f-inverse").

Thus, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1}, and vice versa.

Example 1: Show that each function is the inverse of the other:

$$f(x) = 3x + 8, \text{and } g(x) = \frac{x-8}{3}$$

Finding the Inverse of a Function

The equation for the inverse of a function f can be found as follows:

l. Replace $f(x)$ with y in the equation for $f(x)$

2. Interchange x and y.

3. Solve for y. If this equation does not define y as a function of x, the function f does not have an inverse function and this procedure ends.
If this equation does define y as a function of x, the function f has an inverse function.

4. If f has an inverse function, replace y in Step 3 by $f^{-1}(x)$. We Can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example 2: Find the inverse of the following function

1) $f(x) = 2x + 3$

2) $f(x) = x^3 + 2$

2.7: Inverse Function.

3) $f(x) = (x + 2)^3$

4) $f(x) = x^2$

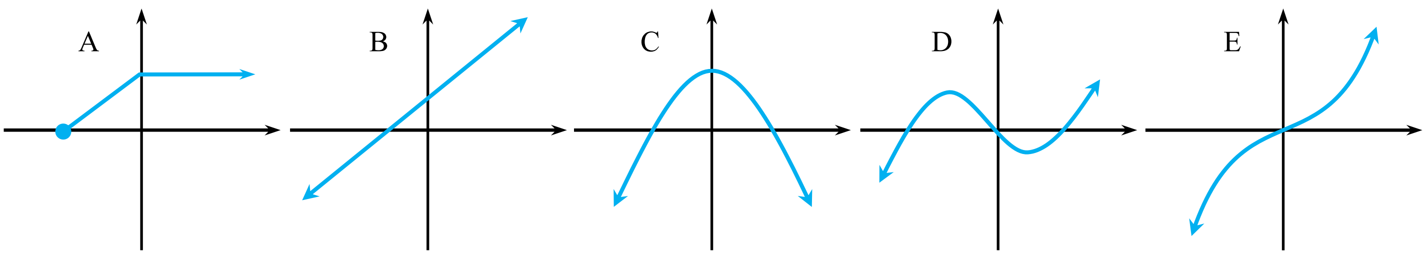
5) $f(x) = x^3 - 2$

6) $f(x) = 5x - 2$

❖ The Horizontal Line Test and One-to- One Functions:

A function f has an inverse that is a function f^{-1}, if the horizontal line intercepts the graph of f exactly at one point.

Example 3: Which of the following graph represent functions that have inverse functions?



2.8: Distance and Midpoint Formula; Circle:

The Distance Formula

The distance, d, between the points, (x_1, y_1) and (x_2, y_2) in the rectangular coordinate system is given by the formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example1: Find the distance between each pair of points.

1) $(5,1)$ and $(8,5)$

2) $(-2,3)$ and $(-5,-2)$

The Midpoint Formula

Consider a line segment whose endpoints are (x_1, y_1) and (x_2, y_2). The coordinates of the segment's midpoint are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example2: Find the midpoint of each line segment with the given endpoints.

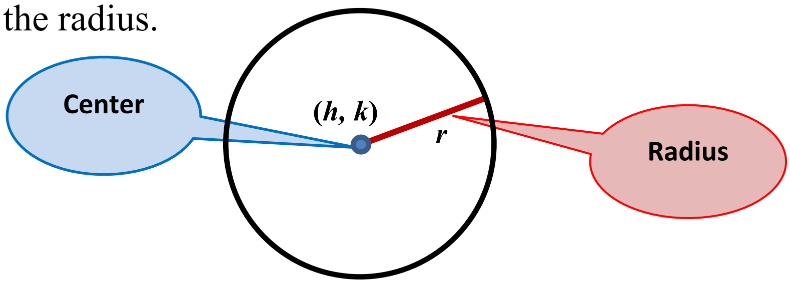
1) $(10,4)$ and $(2,6)$

2) $(-7,-6)$ and $(3,-2)$

2.8: Distance and Midpoint Formula; Circle:

Definition of a Circle

A circle is the set of all points in a plane that are equidistant from a fixed point, called the center. The fixed distance from the circle's center to any point on the circle is called the radius.



The Standard Form of the Equation of a Circle

The standard form of the equation of a circle with center (h, k) and radius r is

$$(x-h)^2+(y-k)^2=r^2$$

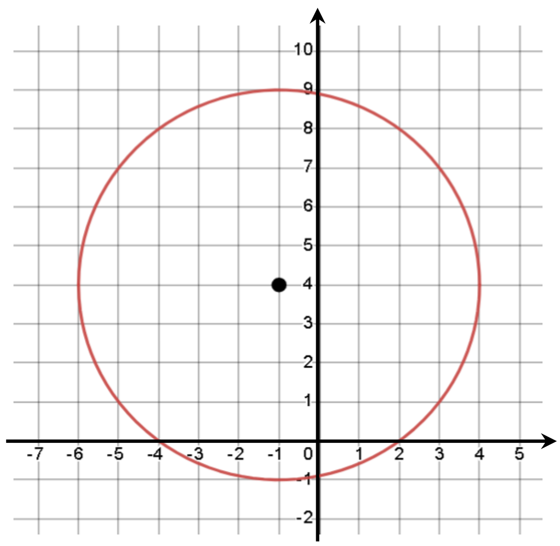
Example3: Write the standard form of the equation of the circle with the given center and radius.

1) Center $(-5,-3), r=\sqrt{5}$

2) Center $(-2,0), r=6$

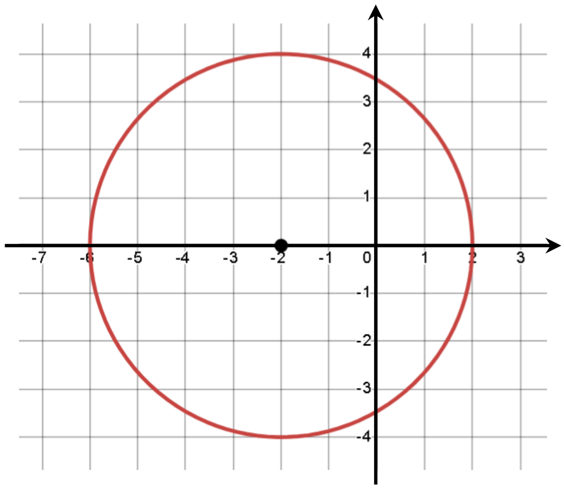
Example4: Give the center and radius of the circle described by the equation and graph equation. Use the graph to identify the relation's domain and range

1) $(x+1)^2+(y-4)^2=25$



2.8: Distance and Midpoint Formula; Circle:

2) $(x+2)^2 + y^2 = 16$



3) $(x-2)^2 + (y-3)^2 = 16$